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DEVELOPMENT OF AN ENGINE/AIRFRAME PERFORMANCE MATCHING SCHEME FOR JET ENGINE RETROFIT

THESIS

Alan Lach, Captain , USAF
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THESIS

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of the Air Force Institute of Technology
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Master of Science in Aeronautical Engineering

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This investigation developed a procedure and new analysis tools for identifying retrofit engine designs for use in existing airframes. The goal was to find a replacement engine for the air-to-air fighter, AAF, developed in the textbook *Aircraft Engine Design*. This paper introduced the ideas of solution surfaces, constraint/thrust diagrams, quality measures and linear regression analysis for the engine design problem. It considered the application of the jet engine design software ACSYS, MISS, and ONX along with statistical analysis software, SAS. Modeling techniques and linear regression analysis were used to minimize iteration while searching a wide range of design variables. Improvements to the design point engine selection process were made through the development of quality measures for choosing appropriate values for bypass ratio and compressor pressure ratio. The analysis tools were assembled into a design scheme for engine retrofit which was then demonstrated with a two-variable and a six-variable design example. The result of the study determined a new baseline engine design for the AAF with increased thrust and decreased fuel consumption. This design process was developed for use by design students in the academic environment.

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Preface

The purpose of this study was to investigate the engine/airframe matching problem as applied to jet engine retrofit. The intent was to produce a design scheme and explore new analysis techniques for use by students studying jet engine and airframe integration. The general design philosophy presented here, builds on the engine design process outlined in the textbook *Aircraft Engine Design* and assumes that the reader is familiar with those procedures and techniques as well as the computer programs used.

The jet engine design problem confronts the engineer with an infinite number of design variations and their effect on aircraft performance. To develop an effective and efficient engine requires the application of a large number of design tools. Past experience, analysis procedures, and computer programs, all aid the designer in determining the best choices among the variations.

In an effort to reduce the amount of iteration involved, I have explored methods to enhance the engine design process as applied to the academic environment. I devised new techniques for decision making and demonstrated their utility. This thesis introduces new methods for measuring improvements and determining trends as well as the application of empirical model building.

Throughout this research, I received support from a number of people whom I would like to take this opportunity to thank. I am indebted to the authors of *Aircraft Engine Design* for their development of the engine/airframe design process which I used as a road map. I greatly appreciate the support and comments provided throughout the research effort by my faculty advisor, Dr. W. C. Elrod, and committee members Dr. P. I. King and Dr. M. E. Franke. I would also like to thank the GSE students with whom I shared an office. Their discussions led me to the use of modeling techniques which significantly improved the design process. Finally, the greatest amount of thanks goes to my wife, Jan, without whose understanding and support this document would not have been produced.

Alan Lach

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List of Symbols

A	-	Area
AB	-	Afterburner
C	-	Specific fuel consumption
C_{DR}	-	Coefficient of additional drags
C_{DRc}	-	Coefficient of drag for drag chute
C_{Do}	-	Coefficient of drag at zero lift
C_L	-	Coefficient of lift
C_P	-	Specific heat at constant pressure
C_{TO}	-	Power takeoff power coefficient
D	-	Drag
e	-	Polytropic efficiency
f	-	Fuel-to-air ratio of burner
f_s	-	Fuel specific work
F	-	Uninstalled thrust; probability distribution
g	-	Acceleration
g_o	-	Local acceleration due to gravity
h	-	Height
h_{PR}	-	Heating value of fuel
K_1	-	Coefficient in lift-drag polar equation
K_2	-	Coefficient in lift-drag polar equation
k_{TO}	-	Velocity ratio at takeoff
L	-	Lift; length
m	-	Mass
m_o	-	Mass flow rate
M	-	Mach number
n	-	Load factor
P	-	Pressure
P_s	-	Weight specific excess power
q	-	Dynamic pressure
$Q_{1 \text{ to } 3}$	-	Quality measures
R	-	Additional drags
S	-	Area; uninstalled thrust specific fuel consumption
T	-	Installed thrust; temperature; probability distribution
TSFC	-	Installed thrust specific fuel consumption
t	-	Time
u	-	Total drag-to-thrust ratio
V	-	Velocity
W	-	Weight
W_E	-	Empty weight
W_F	-	Fuel weight
W_P	-	Payload weight
W_{TO}	-	Takeoff weight
x	-	Independent variable
y	-	Dependent variable

α	-	Installed thrust lapse; bypass ratio
β	-	Instantaneous weight fraction; coefficient in model equation
γ	-	Ratio of specific heats
Δ	-	Change in
ε	-	Error
η	-	Efficiency
θ	-	Static temperature ratio
μ	-	Coefficient of friction
π	-	Total pressure ratio
τ	-	Total temperature ratio
τ_λ	-	Enthalpy ratio of burner
ϕ_{inlet}	-	Inlet external loss coefficient
ϕ_{nozzle}	-	Nozzle external loss coefficient

<u>Subscripts</u>

AB	-	Afterburner
B	-	Braking
c	-	Compressor
c'	-	Fan
FR	-	Free roll
M	-	Mixer
max	-	Maximum
mH	-	Mechanical, high pressure spool
mL	-	Mechanical, low pressure spool
R	-	Rotation
SL	-	Sea-level
t	-	Turbine; total
tH	-	High pressure turbine
tL	-	Low pressure turbine
TD	-	Touchdown
TO	-	Takeoff
0 thru 10	-	Station location

Abstract

This investigation developed a procedure and new analysis tools for identifying retrofit engine conceptual designs for use in existing airframes. The goal was to find a replacement engine with increased sea-level thrust and reduced fuel consumption for the air-to-air fighter, AAF, developed in the textbook *Aircraft Engine Design*. This paper introduced the ideas of solution surfaces, constraint/thrust diagrams, quality measures and linear regression analysis for the engine design problem. It considered the application of the jet engine design software ACSYS, MISS, and ONX along with statistical analysis software, SAS. Modeling techniques and linear regression analysis were used to minimize iteration while searching the maximum range of design variables. Improvements to the design point engine selection were made through the development of quality measures for selecting appropriate values for bypass ratio and compressor pressure ratio. The analysis tools were assembled into a design scheme for engine retrofit. The design scheme was then demonstrated with a two-variable and a six-variable design example. The result of the study determined a new baseline engine design for the AAF with a 16% increase in thrust and a 3% decrease in overall mission fuel consumption. This design process was developed for use by engine design students in the academic environment.

I. Introduction

1.1. Background

Modern aviation ventures require the investment of a great number of resources to introduce a new aircraft. It costs billions of dollars to bring a new aircraft to the market; the failure of a design can be a disaster for the corporation. With so much at stake it is imperative that aircraft designers create the most economical and best performing vehicle possible.

Due to the considerable cost involved in developing entirely new aircraft, many older designs have seen extended service via engine retrofit. Possibly the most obvious use of engine retrofit for existing aircraft is by the military. Almost every aircraft which has endured a long life in military service has done so because of its ability to be updated with better performing engines and equipment. The KC-135 is one example of an old airframe with new engines. The F-15 fighter has seen the use of three different engines over the past twenty years. New engines have taken these aircraft beyond their original design life span and given them new capability and performance.

While the initial airframe and engine design seeks to develop an aircraft with the best blend of design variables, if the aircraft is successful it will most likely become a candidate for engine retrofit in the future. While engine retrofit is a common method of extending the life of an airframe, there is little in the literature that examines this problem in detail. The textbooks to date do not address the retrofit situation specifically, as it is a special case of the overall aircraft design process.

Anyone who has taken an engine design course quickly realizes the enormous task in synthesizing multiple design variables into a viable airframe/engine. There is no single correct answer since design is a continual process of modification, evaluation and compromise.

Engine conceptual design considers the selection of various engine characteristics and design parameters to provide the required thrust while minimizing fuel consumption. The lightest and most cost effective aircraft will be produced by

minimizing the fuel load, since fuel cost is the primary driver of the life cycle cost of an aircraft (12:525).

This design process is iterative, and seeks the best compromise among all variables to achieve the performance goals. The textbook, *Aircraft Engine Design*, was developed to provide a systematic approach to the solution of airframe and engine integration problems.

Along with this approach, the computer programs ACSYS, MISS, ONX and OFFX, References 5, 8 and 9, were developed by Dr. Jack D. Mattingly to facilitate engine design courses in order to alleviate the burden of constant calculation and recalculation for each design option. These programs provide a quick and efficient method for analyzing variations in design parameters and determining their effects on the overall aircraft system.

Airframe and engine matching is essential in the design of an aircraft system. Optimization and trend analyses play a major role in making design choices. Most textbooks concentrate on the airframe design or the propulsion design and stop short of actually performing any optimization or trend analysis. A paper titled, *Airplane Engine Selection by Optimization on Surface Fit Approximations*, Reference 4, outlines the use of some of these optimization methods in the design environment. The method described develops a procedure which can account for numerous variables and optimize designs using surface fit methods (4:595). Engine retrofit is an ideal application for some of the ideas described in the paper.

This thesis examines the engine retrofit problem and the author introduces new methods of data preparation, presentation, and trend analysis to enhance the design process. It introduces the concept of using multivariable linear regression for determining interactions between design variables by developing a link between the overall engine and airframe performance and the component design choices. The computer programs ACSYS, ONX, MISS, and SAS are applied to the retrofit problem.

1.2. Problem Statement

The need exists for a method to identify new jet engine conceptual designs for retrofit in an existing airframe. The development of such a scheme would provide a tool for

engineering students to determine the tradeoffs between various candidate engine designs and select the best choice. The problem proposed requires that a new engine be designed for the air-to-air fighter, AAF, developed in *Aircraft Engine Design.* This is an engine retrofit problem, since the airframe particulars will remain constant while new engine design points are considered to improve the aircraft's overall performance.

1.3. Scope

This investigation developed a procedure for using small sampling and trend analysis in determining the appropriate engine design for retrofit. It covered the engine design process from the selection of design variables to determining the baseline engine. New methods of data presentation and analysis were developed to assist in selecting the best design choices. This study inspired new methods for comparing the overall improvement of each engine design and the use of modeling to reduce the iteration involved in choosing the design point.

1.4. Plan of Development

The jet engine retrofit procedure is developed in the following sections of this paper.

Section 2. The design tools are introduced and tailored to the jet engine retrofit problem. New analysis methods consisting of constraint/thrust diagrams, solution surfaces, quality measures, and modeling are explained. These tools are then assembled into a design scheme which applies the analysis fundamentals and explains the assumptions made.

Section 3. A two-variable and six-variable design analysis is performed to demonstrate the usefulness of the design scheme in selecting alternative design point engines for the AAF fighter. The candidate engine designs are compared and the best design selected as the new baseline.

Section 4. Conclusions are presented and recommendations made for improving the design process.

II. Analysis Fundamentals

This section develops the background information needed to perform the retrofit engine design analysis. It explains the new methods and procedures developed during this research effort and assembles them into a retrofit design scheme.

2.1. Engine and Airframe Analysis

Successful engine design requires matching an appropriate engine cycle to an airframe in order to achieve the desired mission performance from the aircraft. This performance is usually specified by a mission profile and some maximum performance requirements, such as the maximum Mach number or minimum cruise range. The mission may consist of takeoff, climb, cruise, descend, and land as in the case of an airliner, or be much more complicated, including high speed turns, supersonic accelerations and weapons delivery, as in the case of a modern fighter. Once the flight characteristics of the aircraft have been determined the goal is to develop an engine design which will produce adequate thrust throughout the mission flight envelope.

2.2. Constraint Analysis

The constraint analysis allows the designer to determine the appropriate relationship between an aircraft's wing loading and thrust loading based on the required performance. The thrust loading is the ratio of the sea-level thrust to the aircraft takeoff weight (T_{SL}/W_{TO}). The wing loading is the ratio of the takeoff weight to the wing area (W_{TO}/S).

Depending upon the mission, maximum performance objectives, and the constraints placed on the aircraft, a design will emerge which is the best compromise of the wing and thrust loading. The basic aircraft performance requirements and the effect of wing loading and thrust loading have been summarized below.

Airfield Performance	Landing and takeoff considerations are driven by the lifting ability of the wing. A low wing loading is desirable to obtain reasonable landing distances. The thrust loading and wing loading will determine the takeoff distance of the aircraft.
Subsonic cruise and loiter	The distance an aircraft is required to fly and loiter is of major importance for some missions. The contributing factor for an aircraft's range is its wing loading. Medium wing loading is desirable.
Subsonic sustained turn rate	The turn rate is limited by balancing the thrust and drag for the maneuver. A low wing loading results in the best performance.
Instantaneous turn rate	Instantaneous turn rate is limited by the maximum lift attainable. A low wing loading is necessary.
Supersonic maneuvering	Relatively large wing area of low aspect ratio is desirable.
Max Mach number	Tendency is toward lower wing loading for modern fighter aircraft.
Subsonic specific excess power (SEP)	A measure of an aircraft's ability to increase energy (rate of climb) rapidly, (T-D)V/W. The thrust to weight ratio is the dominating factor.

The various factors influencing the choice of wing and thrust loading are not complementary. The conditions required for the airfield, cruise and loiter, and

maneuverability are in opposition to those for maximum speed and low altitude strike. An aircraft designed for high maneuverability will have low wing loading and high thrust to weight ratio, whereas a low altitude bomber will exhibit the opposite trend (14:38-39, 12:84-100).

In order to arrive at reasonable estimates for the wing loading and thrust loading, a relationship between these two design parameters can be developed. This relationship may be used to produce constraint diagrams for determining the combinations of wing loading and thrust loading needed to perform the required maneuvers.

2.2.1. Constraint Diagram Generation

For an aircraft in flight, a force balance can be used to develop an equation which relates T_{SL}/W_{TO} and W_{TO}/S. The generalized force diagram for an aircraft is presented in Figure 1, where D+R are the drag components, L is the lift, W is the aircraft weight, and T is the installed thrust.

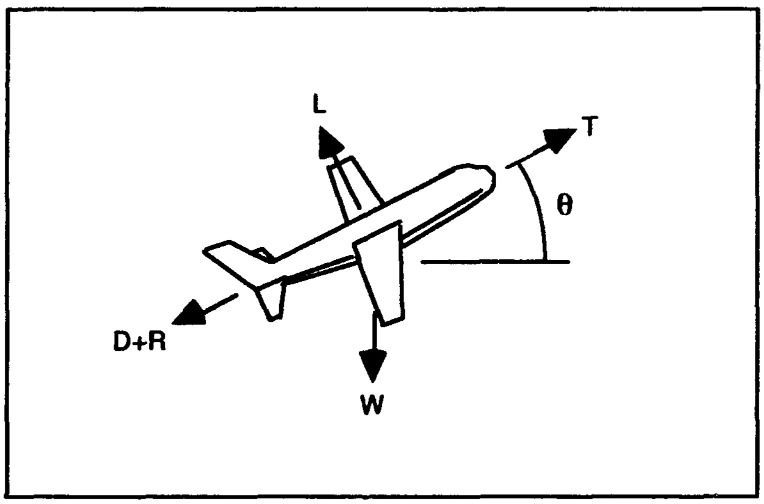


Figure 1. Aircraft Force Diagram

The summation of forces along the flight path is

$$T-(D+R)-W\sin\theta-\frac{W}{g_o}\frac{dV}{dt}=0 \tag{1}$$

where the last term represents the force due to the aircraft's acceleration along the flight path. The aircraft's change in altitude with respect to time can be written as

$$\frac{dh}{dt} = V \sin\theta \tag{2}$$

Solving Eq (2) for $\sin\theta$ and substituting into Eq (1), the force balance becomes

$$T - (D + R) - W\frac{1}{V}\frac{dh}{dt} - \frac{W}{g_o}\frac{dV}{dt} = 0 \tag{3}$$

Multiplying Eq (3) through by V and rearranging results in

$$\{T - (D + R)\}V = W\frac{dh}{dt} + \frac{1}{2}\frac{W}{g_o}\frac{dV^2}{dt} \tag{4}$$

The term to the left of the equals sign represents the rate of mechanical energy added to the system while the first term on the right is the potential energy and the second term is the kinetic energy of the system.

Eq (4) forms the basis for developing the various performance constraints for the aircraft at each condition in the flight. The thrust and weight can be corrected to T_{SL} and W_{TO} by defining the installed thrust ,T, as

$$T = \alpha T_{SL}$$

where α is the installed full throttle thrust lapse, and the instantaneous weight, W, as

$$W = \beta W_{TO}$$

where β is the weight fraction. Eq (4) can be rewritten using α and β along with the traditional lift and drag definitions to arrive at the working form of the equation

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{qS}{\beta W_{TO}}\left[K_1\left(\frac{n\beta}{q}\frac{W_{TO}}{S}\right)^2 + K_2\left(\frac{n\beta}{q}\frac{W_{TO}}{S}\right) + C_{DO} + \frac{R}{qS}\right] + \frac{1}{V}\frac{d}{dt}\left(h + \frac{V^2}{2g_o}\right)\right\} \tag{5}$$

This equation provides the functional relationship between T_{SL}/W_{TO} and W_{TO}/S and can be modified for the various flight conditions. The complete development of this equation can be found in Reference 6.

The last term in Eq (5) includes the weight specific excess power and is defined as

$$P_s = \frac{d}{dt}\left\{h + \frac{V^2}{2g_0}\right\}$$

P_S is a measure of the instantaneous change in potential and kinetic energy of the aircraft and combines the dynamic effects of both climb and acceleration. Using Eq (5), the following flight conditions have been developed in Reference 6 for the constraint analysis.

Constant Altitude/Speed Cruise (P_S=0)
Constant Altitude/Speed Turn (P_S=0)
Constant Speed Climb (P_S=dh/dt)
Service Ceiling (P_S=dh/dt)
Horizontal Acceleration (P_S=VdV/$g_0$dt)
Takeoff Ground Roll (with or without obstacle)
Braking Roll (with or without obstacle)

These flight conditions form the basis of the ACSYS computer program, Reference 5, which allows for easy generation of constraint boundary diagrams. These constraint diagrams are generated in order to determine the values for the wing and thrust loading which lie within the solution space.

Using ACSYS, Figure 2 was produced and shows a completed constraint analysis diagram. The region indicated as the solution space, bounded by the landing, M=0.9 turn, and supercruise constraint lines, represents all possible combinations of wing loading and thrust loading which will successfully perform all the maneuvers specified. More information on the program ACSYS is presented in Appendix A.

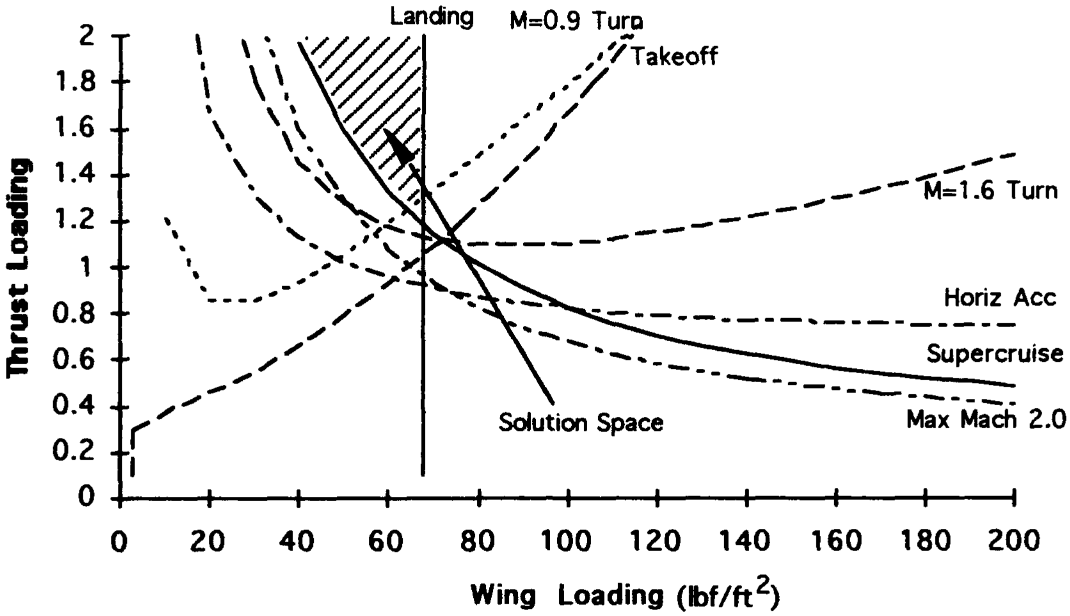


Figure 2. Constraint Analysis Diagram

2.2.2. Constraint/Thrust Diagrams for Engine Retrofit

The constraint diagram was transformed from a device for selecting the design point wing and thrust loading into a method for evaluating various engines that are required to perform specific maneuvers. Constraint/thrust diagrams were developed to provide a means to quickly compare the performance advantages of one engine over another. This was possible due to the fixed wing area which allows lines of constant T_{SL} to be plotted on the constraint diagram.

For the case of an engine retrofit problem, the aircraft particulars are known and can be used to help construct a wing and thrust loading plot for values of constant thrust. A relationship between T_{SL}/W_{TO} and W_{TO}/S was developed and can be represented by the equation

$$\frac{T_{SL}}{W_{TO}} = \frac{T_{SL}}{S\frac{W_{TO}}{S}}$$

Since the aircraft's wing area, S, is known, for any given T_{SL}, T_{SL}/W_{TO} can be evaluated at the various values of W_{TO}/S. Constant thrust lines can be plotted for the values of T_{SL} in which the designer is interested. This resulted in the plot of Figure 3, which presents lines of constant thrust based on an aircraft's wing area of 381 ft^2.

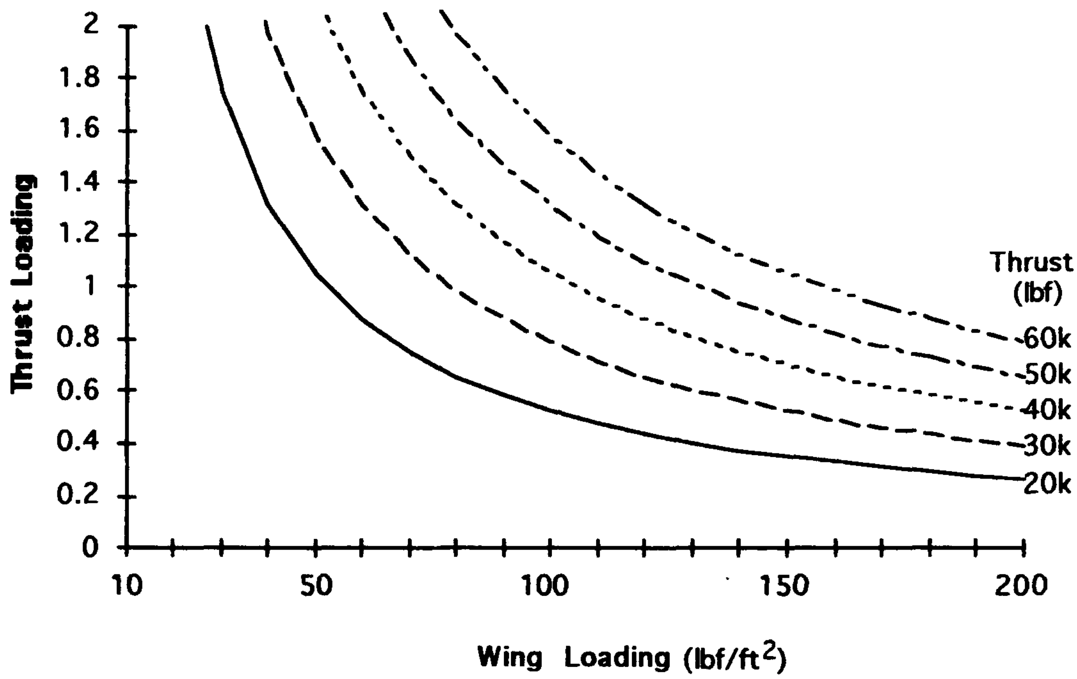


Figure 3. Thrust Loading vs Wing Loading and Lines of Constant Thrust

When a particular flight maneuver or condition is desired, a constraint diagram for various values of β can be created. For example, if the aircraft is required to negotiate a M=0.9, 5g turn at 30,000 ft, the thrust loading and wing loading requirements at various values of β are plotted as shown in Figure 4.

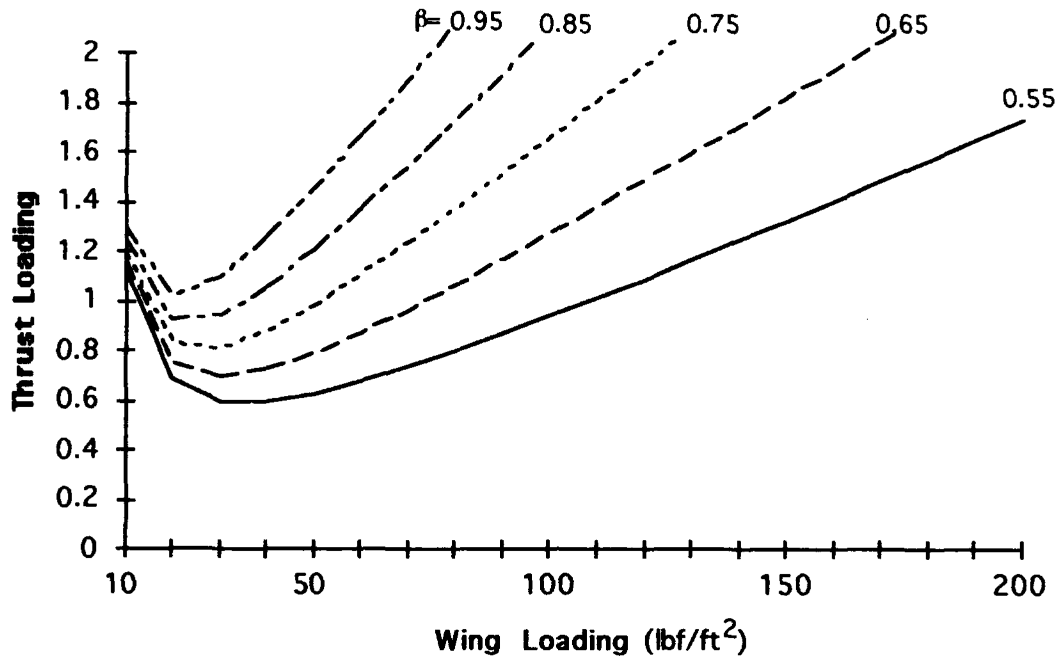


Figure 4. Thrust Loading vs Wing Loading M=0.9, 5g Turn, 30k ft

By combining Figures 3 and 4, a new diagram called the constraint/thrust diagram was created. Figure 5 presents the constraint/thrust diagram and shows the constraint boundary for various β values along with the lines of constant thrust. The solution boundaries of this chart are read below the lines of constant T_{SL}. All combinations of β, wing loading and thrust loading are solutions if they lie below the thrust curve. Any combination above the thrust curve will not be possible. From this diagram the minimum sea-level thrust needed for negotiating the turn at various weight fractions can be determined.

For example, if the desire is to fly an aircraft with a wing loading of 90 and a thrust loading of 0.9 while negotiating this turn at β=0.55 then the required T_{SL} would be 30,000 lbf. This would be the design goal, to develop an engine or engines to produce 30,000 lbf of installed thrust at sea level.

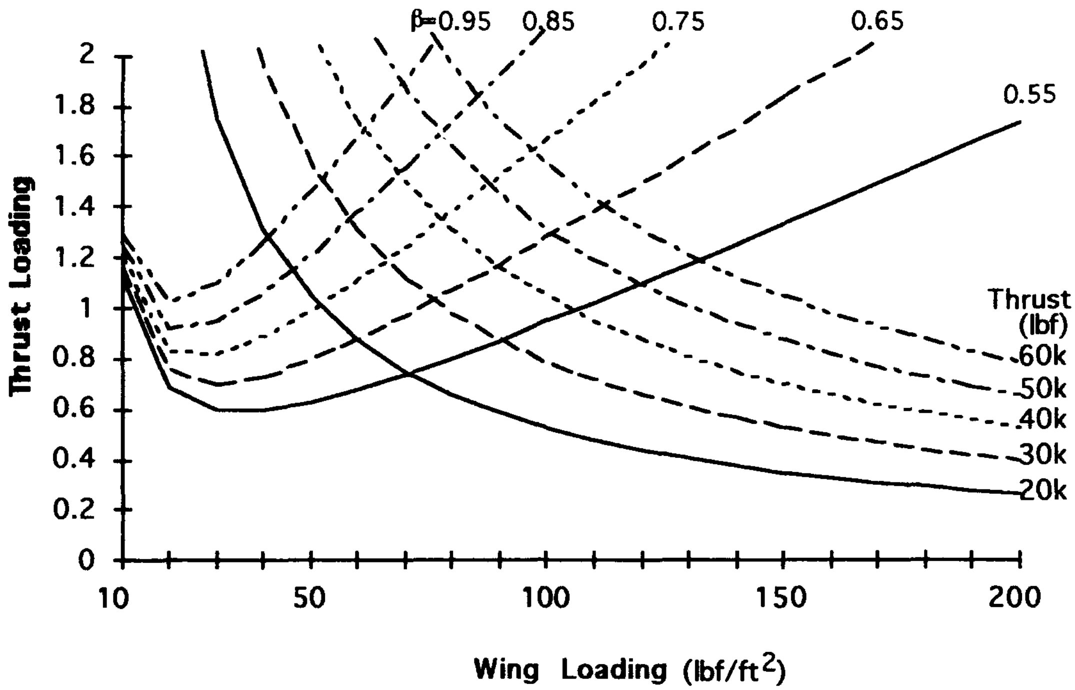


Figure 5. Constraint/Thrust Diagram, M=0.9, 5g Turn, 30k ft

This plot can also be used to determine the range of performance of an aircraft if the sea-level thrust is known. Using Figure 5, if the available thrust is 30,000 lbf and the wing loading is 70, the aircraft will only be able to negotiate a M=0.9 turn for weight fractions below approximately 0.7. With the lines of constant T_{SL} plotted, any engine whose T_{SL} is known, can be evaluated to determine the aircraft weight fraction at which the various maneuvers can be performed.

For engine retrofit, the original aircraft wing loading is kept the same, since it is based on the aircraft's takeoff weight (the retrofit engine weight change is ignored) and wing area which are known. The desire is to move along a constant wing loading line with the various engine designs and evaluate their performance.

This concept can be extended to evaluate new takeoff weights and wing loading for the retrofit aircraft. Since all combinations of wing loading, thrust loading and β below the constant thrust line are possible, new combinations may be selected which will allow performance at higher β values.

The constraint/thrust diagram converts the constraint diagram into a flexible analysis tool upon which to place the attributes of each engine design and analyze its effectiveness for performing various maneuvers. As the retrofit aircraft performance is being determined, the designer can shift the wing loading to gain insight into the maneuverability of the aircraft at various takeoff weights and weight fractions. This analysis allows the aircraft designer to compare new engine designs to see how the changes in T_{SL} will affect each maneuver and not just a specific mission.

2.3. Mission Analysis

The mission analysis portion of the design process, as developed in chapter 3 of Reference 6, uses the wing and thrust loading values selected from the constraint diagram to determine an acceptable takeoff weight for the aircraft.

The aircraft weight is simply a combination of the empty weight, fuel weight and payload weight

$$W_{TO}=W_E+W_F+W_P$$

The rate of change of the aircraft weight as a function of the fuel consumption is

$$\frac{dW}{dt} = -\frac{dW_F}{dt} = -TSFC \times T$$

where TSFC is the installed thrust specific fuel consumption and T is the installed thrust. This equation can be rewritten as

$$\frac{dW}{W} = -TSFC\frac{T}{W}dt \qquad (6)$$

The desire is to determine appropriate expressions for various legs of the mission which relate the initial and final weights for that phase. These expressions are of the form

$$\frac{W_f}{W_i} \leq 1$$

and Eq (6) is used to develop the weight fraction equations which correspond to flight regimes where $P_S>0$ (type A), or $P_S=0$ (type B). Type A behavior is exhibited during constant speed climb, horizontal acceleration, climb and acceleration, and takeoff acceleration. Type B behavior relates to constant altitude/speed cruise and turn, best cruise Mach number and altitude, loiter, warmup, takeoff rotation, and constant energy height maneuvers. The developed equations are

Type A:

$$\frac{W_f}{W_i} = \exp\left\{-\frac{C\sqrt{\theta}}{V(1-u)}\Delta\left(h+\frac{V^2}{2g_o}\right)\right\}$$

Type B:

$$\frac{W_f}{W_i} = \exp\left\{-C\sqrt{\theta}\left(\frac{D+R}{W}\right)\Delta t\right\}$$

Where C is the specific fuel consumption, θ is the static temperature ratio, and u is the total drag-to-thrust ratio. These equations are used in the computer program MISS, Reference 8, to calculate the weight fractions for any given mission leg. More information on the MISS computer program is pro' .ded in Appendix B.

For the engine retrofit problem, the design takeoff weight is known and the mission analysis is used to evalute the overall fuel consumption of each new engine design.

2.4. On-Design Analysis

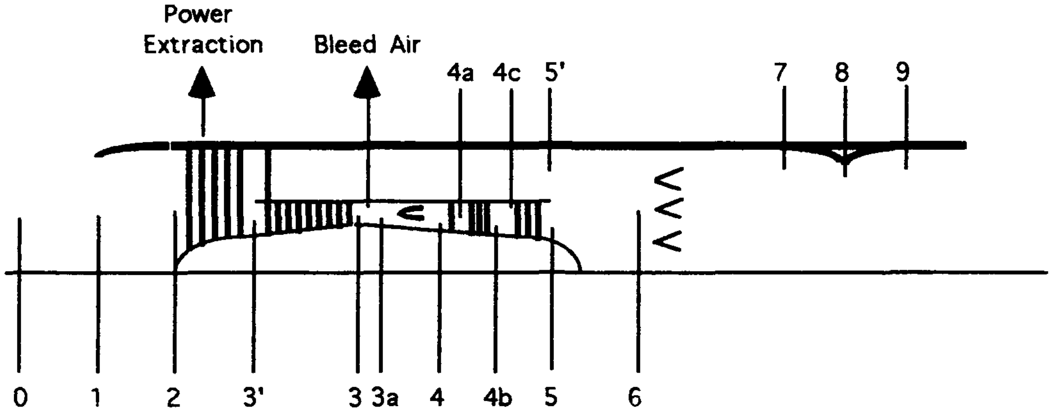
2.4.1. On-Design Solution

The on-design cycle analysis was used to select a reasonable range for the engine design variables at the design point. This analysis, which will be referred to as the parametric analysis in future papers by Mattingly and Heiser (7:2), allows the engine designer to select a design point and calculate engine performance. In particular, values for the two most important design characteristics at this point, the uninstalled specific thrust, F/m_0, and the uninstalled thrust specific fuel consumption, S, can be determined. The engines developed with the on-design analysis are referred to as the design point engines.

In order to calculate F/m_0 and S, a closed form solution for the on-design analysis can be achieved by solving the series of equations presented in Appendix E of Reference 6. The inputs and outputs to these equations are presented in Table 1. Figure 6 illustrates the reference station numbering for a mixed-stream turbofan engine. Each reference station number pertains to the locations of the mass flow as it migrates through the engine.

To begin the analysis, the design altitude and Mach number are selected. These values are usually based on that leg of the mission which is the most demanding or best encompasses the range of mission requirements. Judgement is required when selecting this starting point but some general comments can be made.

If an aircraft will subsonically cruise for long distances, then the engine performance at the cruise altitude and Mach number is going to be most important. Fuel economy should be the best at this cruise condition for the engine design. On the other hand, if the design calls for an aircraft which must perform at a high Mach number and supercruise then the design altitude and Mach number should reflect the best blend of these conditions (6:129)



Station	Location	Station	Location
0	Far upstream	4b	High pressure turbine exit
1	Inlet or Diffuser	4c	Low pressure turbine entry
2	Inlet or diffuser exit Fan entry	5	Low pressure turbine exit Mixer entry
3'	Fan exit High pressure compressor entry	5'	Fan bypass mixer entry
3	High pressure compressor exit	6	Mixer exit Afterburner entry
3a	Burner exit	7	Afterburner exit Exhaust nozzle entry
4	Burner exit Nozzle vanes entry	8	Exhaust nozzle throat
4a	Nozzle vane exit High pressure turbine entry	9	Exhaust nozzle exit

Figure 6. Engine Reference Stations (6:99)

Table 1. On-Design Solution Inputs and Outputs (6:467)

	Inputs
Flight parameters:	M_0, T_0(R), P_0(psia)
Aircraft system parameters:	β, C_{TO}
Design limitations:	
Perfect gas constants:	γ_c, γ_t, γ_{AB}, C_{pc}, C_{pt}, C_{pAB} (BTU/lbm-R)
Fuel heating value:	h_{PR} (BTU/lbm)
Component figures of merit:	$\varepsilon 1$, $\varepsilon 2$
	π_b, π_{dmax}, π_{Mmax}, π_{AB}, π_n
	e_c', e_{cH}, e_{tH}, e_{tL}
	η_b, η_{AB}, η_{mL}, η_{mP}, η_{mH}
Design choices:	π_c', π_c, α, T_{t4} and T_{t7} (°R), M_5, P_0/P_9
	Outputs
Overall performance:	F/m_0 (lbf/lbm/s), S (1/h), f_0,
	η_P, η_{TH}, V_9/V_0, P_{t9}/P_9, T_9/T_0
Component behavior:	π_{tH}, π_{tL}, π_M,
	τ_c', τ_{cH}, τ_{tL}, τ_{tH}, τ_λ, $\tau_{\lambda AB}$
	f_{AB}, f
	η_c', η_{cH}, η_{tH}, η_{tL}
	M_5', M_6, M_9

Once the altitude and Mach number have been selected, the on-design analysis is used to generate solution surfaces and trend plots from which to select the best design point engine values for further analysis. At this point in the design scheme the engine is considered to be a "rubber engine" with variable size and thrust which can be stretched and shaped to fit the performance requirements (12:101).

The computer program ONX provides an efficient tool for generating the on-design data. A complete description of this program can be found in Reference 9.

2.4.2. Carpet Plots and Solution Surfaces

The on-design calculations produce data used to generate plots of F/m_0 and S based on various combinations of design variables. These plots are used to select reasonable ranges for the design variables by seeking those combinations which maximize F/m_0 and minimize S. The most useful representation presents the effect of the compressor pressure ratio, π_c, and the bypass ratio, α, on F/m_0 and S. This results in a typical carpet plot as shown in Figure 7, where each data point represents the value of F/m_0 and S at a particular combination of π_c and α.

Carpet plots are produced for various combinations of Mach number, altitude, turbine inlet temperature, etc. and are used to select a suitable range for the compressor pressure ratio and bypass ratio. This is done by looking below the knee of the constant α curves and choosing the π_c and α values which will reduce S while minimizing the reduction in F/m_0.

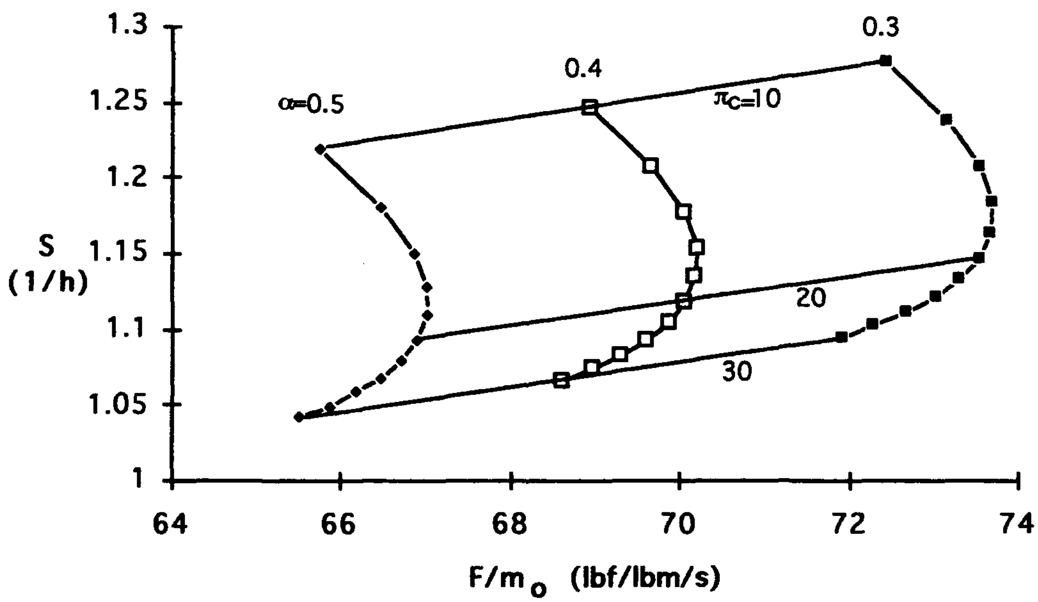


Figure 7. Carpet Plot, M=0.9, h=35 k ft, T_{t4}=3200 R

The mixer of a mixed stream turbofan engine plays an important role in the engine's overall performance. The Mach number of the airflows leaving stations 5 and 5' influence the temperatures and pressures of all the rotating components and must

be compatible to allow for proper mixing at station 6. Since M_5 and M_5' will change for various flight conditions, it is prudent to select only those combinations of π_c and α for which M_5 and M_5' lie between 0.4 and 0.6 at the design point (6:123). The results presented in the carpet plots are therefore misleading, since the plot presents a large number of compressor pressure ratios and bypass ratios where a useful solution does not exist.

Using ONX, a series of on-design calculations were made by varying π_c from 6 to 30 and α from 0.2 to 0.6 for M=1.5 at an altitude of 35,000 ft and T_{t4}=3200 R. After extracting those combinations of π_c and α which result in an M_5 and M_5' between 0.4 and 0.6, the carpet plot shown in Figure 8 was generated. Unlike the carpet plot of Figure 7, there are large gaps in the useful combinations of π_c and α. There is no value for π_c which will extend from the lowest α to the highest and include all α values. Also, at this design point, the lines of constant α do not form a knee. These conditions make selecting design point ranges for π_c and α more difficult.

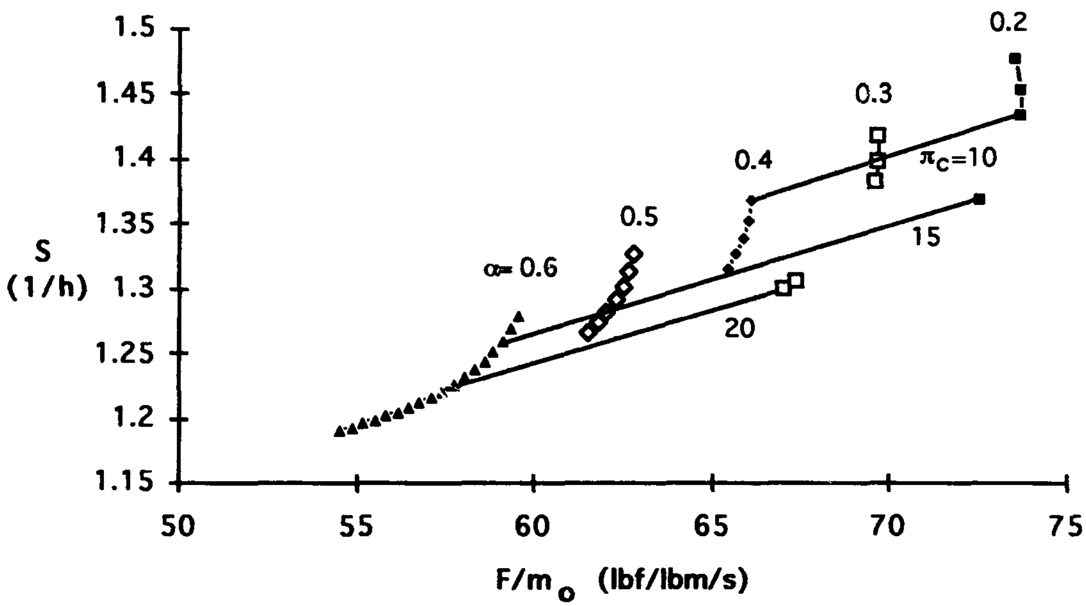


Figure 8. Carpet Plot, M=1.5, 35k ft, T_{t4}=3200 R,
M_5 and M_5' between 0.4 and 0.6

A new method for displaying and interpreting this information was developed through the use of surface plots. For the same on-design condition as used in Figure 8, the F/m_0 and S surface plots have been generated and are presented in Figures 9 and 10. Anywhere the surface is above the arbitrary minimum F/m_0 and S of 50 and 1.2 respectively, and within or along the heavy block border, is a possible combination of π_c and α which should perform well at off-design conditions and can be considered a solution for off-design operation. This plot will be called the solution surface since it represents all acceptable candidate engines for the given design condition and limits. It is this surface which presents the acceptable solutions and graphically demonstrates the influence of the two design variables, π_c and α, on F/m_0 and S.

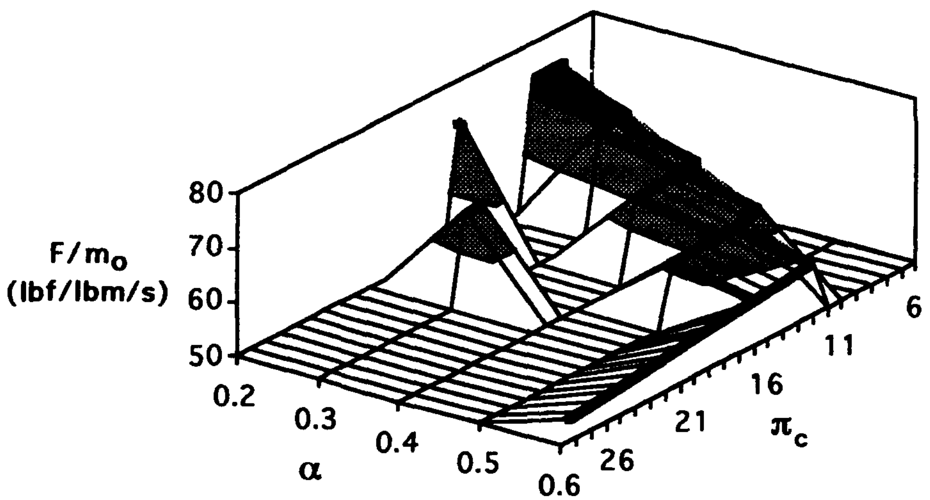


Figure 9. Uninstalled Specific Thrust (F/m_0) Solution Surface

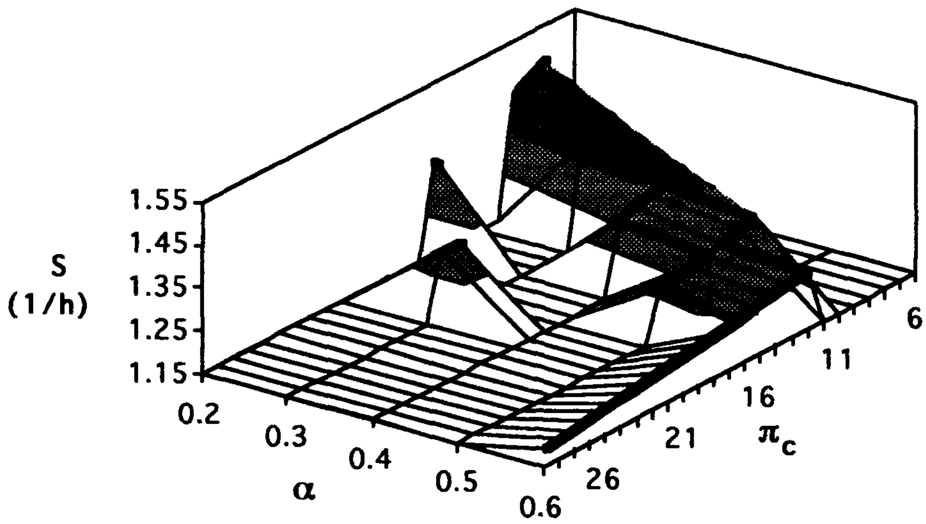


Figure 10. Uninstalled Thrust Specific Fuel Consumption (S) Solution Surface

The impact of the solution surface diagrams is quite striking in that they provide a visual clue as to where the majority of solutions lie. Two points should be made about the solution surfaces. First, the regions which are represented by sharp peaks may or may not contain a reasonable solution. These points emerge when the values for M_5 and M_5' just fall within limits. Second, some solutions may lie off the solution surface where M_5 and M_5' just fall outside the limits. Neither of these conditions can be relied upon to produce a strong candidate engine and the solution surface is meant to represent the best choices of design variable combinations for off-design operation.

To verify the validity of the solution surface analogy and limiting M_5 and M_5', an off-design analysis on all combinations of π_c and α in Figures 9 and 10 was accomplished. If an engine with a particular combination of π_c and α performed successfully at off-design conditions it represents a design point solution and is indicated on Figure 11 by a square. The mixer limited solutions, from the solution surfaces, are represented by a dot on Figure 11.This figure shows that the majority of the mixer limited solutions fall within the region of mixer unlimited solutions. Figure 11 also indicates that a solution does not exist where the spikes were present in Figures 9 and 10 and that the mixer unlimited solutions extend over a slightly larger area.

Since the majority of the mixer limited solutions, 92%, fall within the mixer unlimited region, it may be concluded that limiting the Mach number at station 5 and 5' results in an acceptable number of possible solutions for selecting the design point engine and that the solution surface is a valid representation of this information.

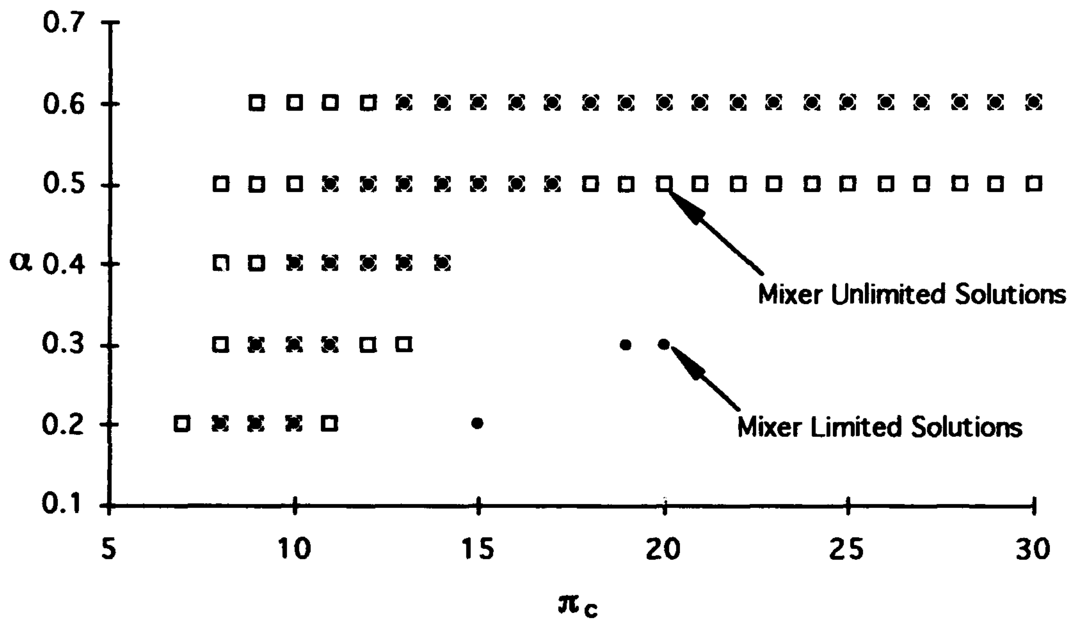


Figure 11. Mixer Limited and Mixer Unlimited Solutions

2.4.3. Generation of Trend Data

The solution surface representation of the uninstalled specific thrust and uninstalled thrust specific fuel consumption provides information as to where possible solutions exist. It is necessary that a measure be developed with which to evaluate the various combinations.

The method developed here is based on the changes in F/m_o and S from one compressor pressure ratio to the next along a line of constant α. For each incremental change in π_c, the fractional change in F/m_o and S can be calculated. The change in F/m_o can be represented by the equation

$$Q_1 = \frac{\left(\frac{F}{m_o}\right)_{i+1} - \left(\frac{F}{m_o}\right)_i}{\left(\frac{F}{m_o}\right)_i}$$

where i varies from 1 to n-1, and n is the number of π_c values evaluated along the line of constant α. A similar equation for S is

$$Q_2 = \frac{S_{i+1} - S_i}{S_i}$$

The quantities Q_1 and Q_2 are defined as quality measures as they establish a characteristic measure for F/m_o and S.

As an example, the quality measures were calculated for the same on-design data represented in Figures 9 and 10 in the previous section. After calculating the various quantities for Q_1 and Q_2, the two measures can be plotted against π_c to generate the plot shown in Figure 12.

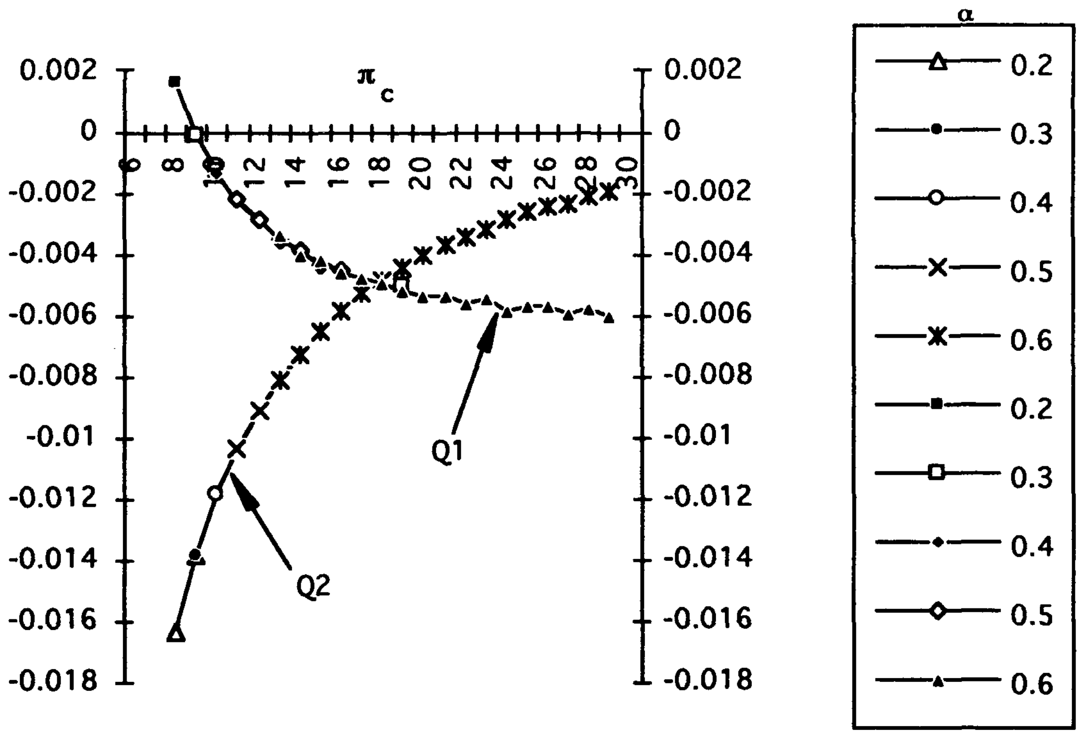


Figure 12. Quality Measures Q_1 and Q_2 for M=1.5, 35 k ft

The negative value on the y-axis, indicates that F/m_0 and S are being reduced for every increment of π_c, a positive value would indicate an increase. For this case, there was an increase in F/m_0 at π_c=8. The rest of the plot shows that there is a steady decline in both F/m_0 and S as the compressor pressure ratio is increased. The decrease in F/m_0 is small at first, as indicated by the small negative values, and grows to -0.006. S starts out with large improvements in fuel consumption at the low pressure ratios and drops to very small gains at the higher pressure ratios. The portions of the curves which tend to flatten out indicate a constant rate of change from one pressure ratio to the next.

A final measure can be developed which presents the two measures above plotted in a single relationship.

$$Q_3 = \frac{Q_1}{Q_2}$$

This quality measure presents the relationship between the fractional change in F/m_0 and the fractional change in S between subsequent values of π_c. If the fractional

change in F/m_0 decreased by 0.1 while S decreased by 0.05 the result is 2, which indicates that F/m_0 decreased more rapidly than the corresponding decrease in S between the two pressure ratios. The plot of Q_3 for the values of Q_1 and Q_2 previously calculated is presented in Figure 13.

This plot indicates that at the high π_c values, F/m_0 decreases more quickly than the corresponding decrease in S. At $Q_3=1$, both values are changing at the same rate. Below 1, S is changing more rapidly, and above 1, F/m_0 changes more rapidly.

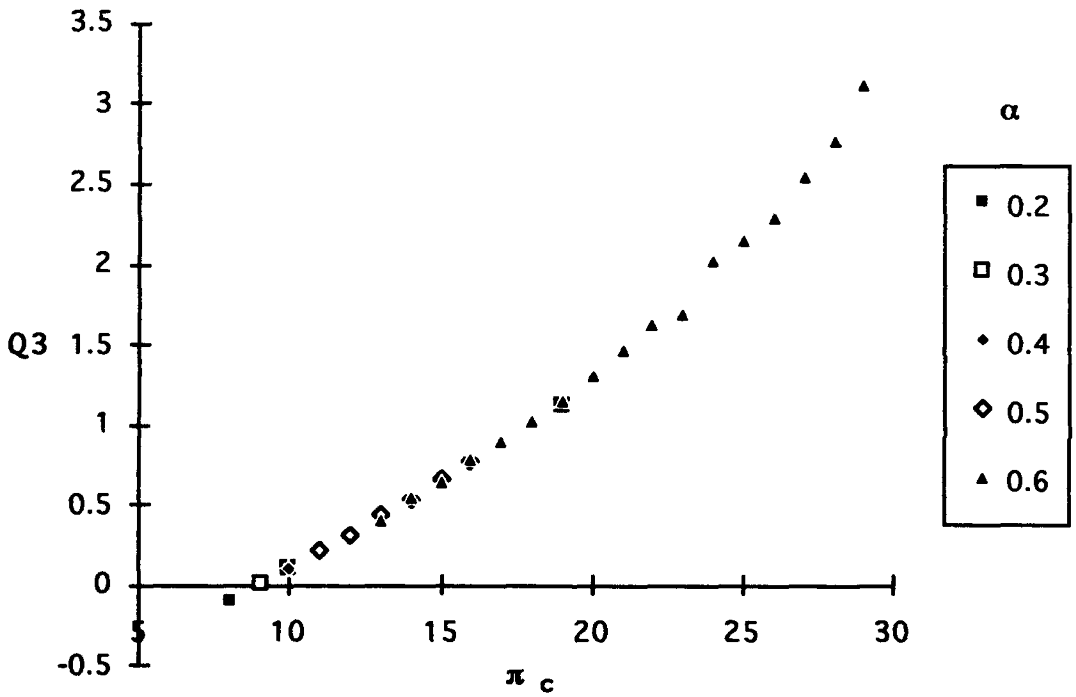


Figure 13. Quality Measure Q_3 for M=1.5, 35k ft

These three quality measures assist the designer by presenting the fractional changes in F/m_0 and S when considering various flight altitudes and conditions for the design point engine. The quality measures are used, rather than the simple difference in F/m_0 or S per π_c increment, in order to allow comparison between the two quantities and across various Mach numbers and altitudes.

For example, plots of the quality measures were created for exactly the same engine parameters as Figures 12 and 13 but at M=1.5 and an altitude of 25,000 ft. The results

are presented in Figures 14 and 15. Figure 14 shows that both F/m_0 and S are changing at the same rate when π_c is 14 whereas the same condition occurs at a π_c of approximately 18 for the 35,000 ft case. This indicates that at the lower altitude the combinations of π_c and α which will minimize S while maximizing F/m_0 begin at lower values of π_c. Also, the intersection for the 25,000 ft case occurs at a higher fraction but a lower π_c than for the 35,000 ft case. Figure 15 indicates that, for any π_c, the decrease in F/m_0 is much higher than the change in S when compared to Figure 13.

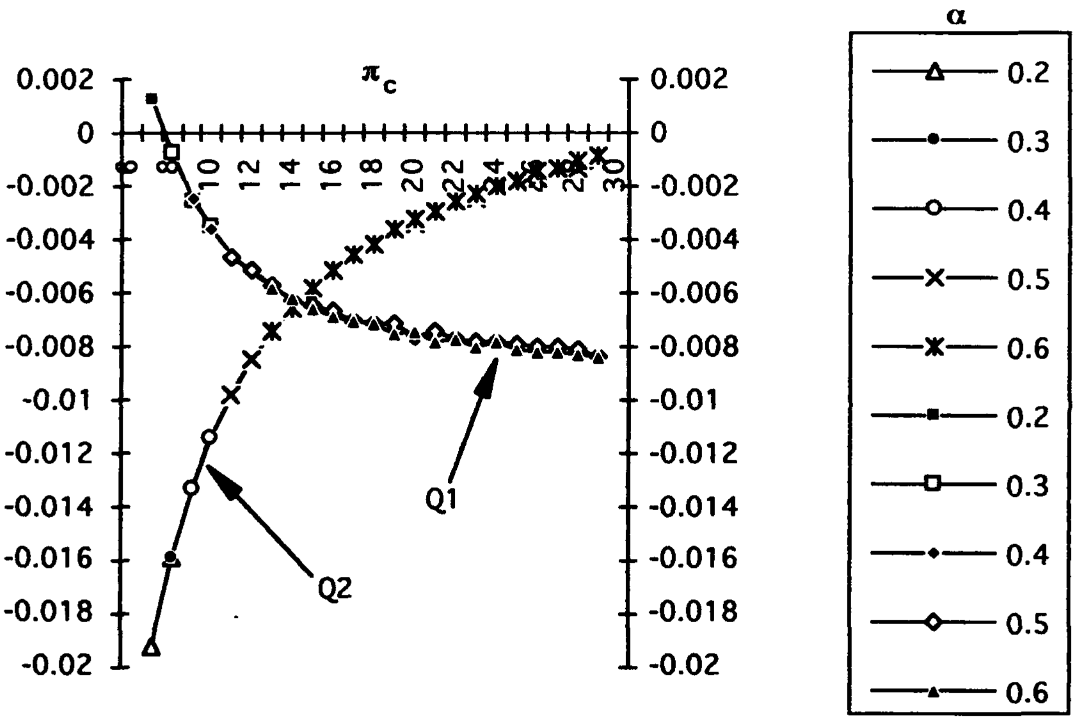


Figure 14. Quality Measures Q_1 and Q_2 for M=1.5, 25 k ft

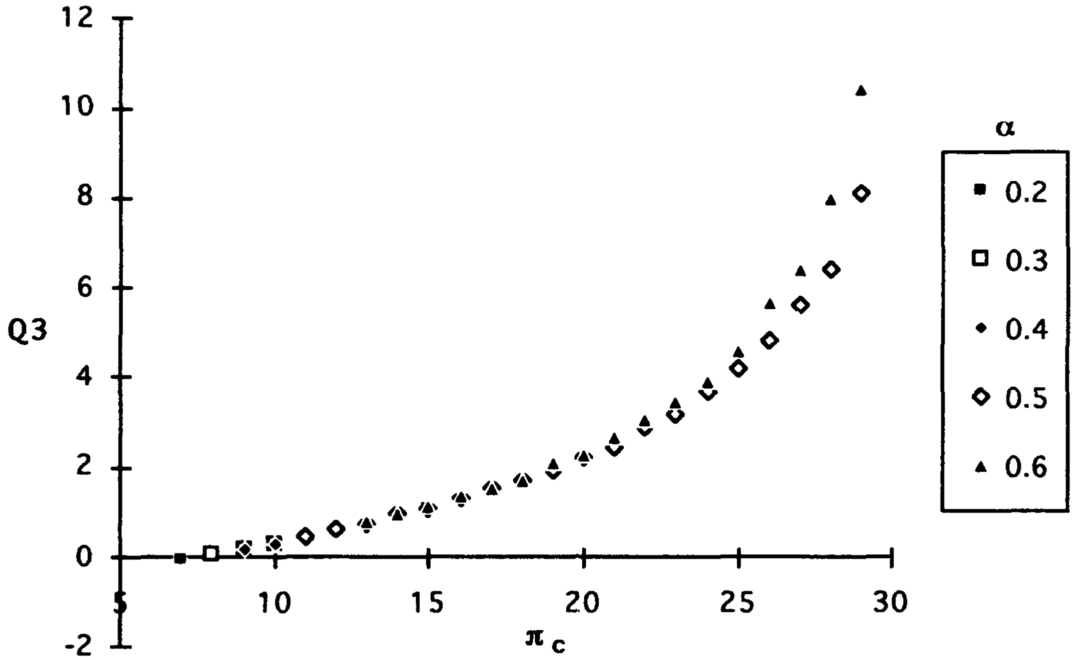


Figure 15. Quality Measure Q3 for M=1.5, 25k ft

Figures 12 and 14 are used to determine the points where the tradeoff between maximum F/m_0 and minimum S is acceptable. For a pure performance application, as in a high performance test vehicle, the choice would be to select a range for π_c and α to the left of the intersection. If fuel economy is the primary concern, the choice would be to move as far to the right of the intersection as possible. For a high performance fighter aircraft, the best of both worlds is desirable, and choices should be made just to the right of the intersection.

Figures 13 and 15 show that the ratio of change in F/m_0 to change in S is much greater for the 25,000 ft case. This would result in selecting a narrower range of π_c values for an investigation of design point engines at 25,000 ft.

These quality measures create an analysis tool for displaying the interactions between F/m_0, S, π_c, and α. This analysis and plot generation was used in the engine retrofit effort to select the range of values for π_c and α which should yield design point engines with good fuel economy and adequate thrust.

2.5. Off-Design Analysis

The off-design cycle analysis provides a means to operate the design point engine at various flight conditions. The solution to the off-design problem is presented in Chapter 5 of Reference 6 and results in 14 dependent variables and 14 independent equations. In order to solve this system of equations, an iterative approach is used. This approach brings all variables within their specified operating limits to calculate the uninstalled thrust, F, and fuel consumption, S. The analysis produces uninstalled engine data over various in-flight operating conditions.

The off-design analysis provides a tool for determining the baseline engine design. The baseline engine is the engine which has been selected due to its ability to successfully complete all mission and performance requirements while minimizing the fuel consumed.

The program OFFX, allows rapid iteration and calculation of the engine performance at off-design conditions. It can be used to make adjustments to the design variables by examining any off-design set of conditions more critically. Additional information on the OFFX program and its various uses can be found in Reference 9.

The off-design analysis is mentioned here because a derivative of it is used in the computer program MISS. MISS is capable of generating off-design performance data for specific mission legs and was used to evaluate each design point engine in an off-design scenario. Additional information on MISS is presented in Appendix B.

2.6. Modeling Techniques

In order to reduce the amount of iteration necessary when developing new design point engines, empirical modeling was used. The purpose of empirical modeling is to take a related group of predictor values and generate a response to them. A group of independent variables or input values is developed into a model which predicts the dependent variable or output value. The method of multivariable linear regression and the application of statistical analysis for empirical model building will be discussed here.

If the response to a given set of inputs can be demonstrated by a simple relationship it is referred to as a deterministic or mechanistic model. For example, if the response to an input, x, follows the rule

$$y = 3x + 6$$

it is a mechanistic model since it predicts with very little error and it is a simple matter to predict the outcome of any input, x (1:11). When systems become more complex and their behavior cannot be modeled using a simple equation, then other means must be used to achieve a useful predictor of future outcomes.

The creation of a model which relies on observations or experience without considering the theoretical aspect of the analysis, is the foundation of empirical modeling. Each observation of the response to a certain set of input variables is an experiment. The prediction of other outputs can be made based on careful analysis of the results. Of interest here are functions of more than one variable, where

$$y = f(x_1, x_2, \ldots x_k)$$

If each of the x_i's are thought of as vector components , then this equation can be simply stated as

$$y = f(\mathbf{x})$$

where $\mathbf{x}$ is a one dimensional column vector representing the input variables. Usually, a set of physical parameters, $\theta_1, \theta_2, \ldots, \theta_p$, define this functional relationship between the input and output. The result is

$$y = f(\mathbf{x}, \boldsymbol{\theta})$$

If the functional relationship linking the inputs to the output are precisely known, a mechanistic model is formed. When the relationship between the input and output is unknown, the response may be approximated if the functional relationship is assumed

smooth. This relationship can be written as

$$y \approx f(\mathbf{x}, \boldsymbol{\beta})$$

where the vector $\boldsymbol{\beta}$, refers to the approximate values of the physical parameters $\boldsymbol{\theta}$. Thus, the empirical model relates the input to the output through an approximation. Since the result of this calculation is an approximation, there is some error, ε, which is a random variable associated with the particular set of independent variables. The linear model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k + \varepsilon$$

relates the behavior of the dependent variable y to the independent variables x_j's with an error term ε.

In matrix notation, letting y_i, x_{ij}, and ε_i be the values for y, x_j, and ε in the i^{th} experiment, and defining the Y vector, the X matrix, the $\boldsymbol{\beta}$ vector, and the ε vector as

$$\mathbf{Y} = \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1k} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ 1 & x_{n1} & \cdots & x_{nk} \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \cdot \\ \cdot \\ \cdot \\ \beta_k \end{bmatrix}, \quad \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_n \end{bmatrix}$$

the model becomes

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

where the number of experiments or observations is n and the number of variables is k.

After experiments are performed using various input levels for the independent variables, the information can be used to develop an approximation to the actual function. Despite the fact that this is an approximation, a well developed model allows general statements to be made about the interactions between the input variables and assess their impact on the output.

After making n observations of the y variable using k independent variables, estimates for the values of $\boldsymbol{\beta}$ can be made. The resulting equation is

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \ldots + \hat{\beta}_k x_k$$

The least squares estimate for $\hat{\boldsymbol{\beta}}' = (\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_k)$, is found by solving

$$\hat{\boldsymbol{\beta}} = (\mathbf{X'X})^{-1}\mathbf{X'Y}$$

After solving for the values of $\hat{\boldsymbol{\beta}}$, the model must be examined to see how well it conforms to the data being modeled. The process is an iterative one using various statistical methods for determining how well the model fits the data (1:34-35, 3:9, 10:493-500).

When modeling, there is no requirement that all the terms in the input have to be used in the model, since some of the terms may not produce significant changes in the result. The terms can be second order, third order, exponential, etc., as long as the model accurately characterizes the data within the region of interest (2:218-224). Second order approximations of the form

$$g(\mathbf{x}, \boldsymbol{\beta}) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{12} x_1 x_2$$

were used to develop the models in this investigation.

The computer program SAS, Reference 13, was used extensively for creating the models, determining how well they fit the data, and making predictions for the sea-level thrust and the mission fuel consumption. How suitably the model fits the data can be determined using statistical analysis techniques. The specific tests used for linear regression as applied to the SAS environment are discussed in Appendix C on SAS programming.

2.7. Engine Retrofit Design Scheme

The engine retrofit problem differs from the aircraft and engine design problem outlined in Reference 6, in that the goal was to find a new baseline engine which improved the performance of an existing aircraft design. If this was a new aircraft development program, the best engine and airframe integration would be produced using any of the procedures outlined in References 6, 11 or 12, since both the airframe and engine design variables are flexible in the design process. By keeping the airframe constant , the design scheme can be simplified to a certain degree and new analysis and comparison methods demonstrated.

The two major measures of improvement for the retrofit problem were the overall fuel consumption and the available sea-level thrust. The search can be for improved fuel economy while maintaining the same thrust, increased thrust without regard to fuel economy, or search for improvement in both thrust and fuel economy. For the first case, the amount of fuel which the aircraft will have to carry will be reduced. The benefit is increased range or increased payload equal to the reduction in fuel weight. The second case relates to a pure performance improvement, as in a high performance test application, where fuel economy is not a concern. The final case results in an overall improvement in performance across the board. This case is applicable to a fighter type aircraft and was the focus of this investigation.

The problem was to search for trends and make inferences by testing a minimum number of alternative engine designs. The procedure outlined in this section presents the process used for selecting appropriate engine design parameters when designing a new engine for an existing airframe. It incorporates existing computer programs to generate performance data while developing methods to weigh the various outputs in a manner which will lead the investigator toward a feasible design.

Items A through H below, describe the analysis approach developed in this investigation. They present a brief description of the process along with the assumptions and methods involved in its use. The technique described in Reference 6, was very useful as a road map in planning the retrofit analysis.

A. Assemble the design point information.

The existing engine and airframe have a design point. This is where the initial information for the search was found. Since the retrofit aircraft was known, information on the drag characteristics, the effective wing area, and the design thrust loading and wing loading was available. This information was used to develop the baseline airframe upon which to hang candidate engines and evaluate their performance.

The allowable space for the engine is fixed and the engine inlet sized by the restriction of the fuselage. It was assumed that the existing inlet area and the length and diameter of the nozzle would remain constant.

With the inlet area determined, the mass flow rate is limited to a maximum for each engine design. This is critical in the search for the new engine since the performance at off-design flight conditions depends on the available airflow.

The existing baseline engine was the first source of information for defining the new engine design. It already performs all aspects of the mission and provides acceptable fuel economy. The retrofit engine design may incorporate changes for component efficiencies, fuel heating value, or any of the numerous input design variables for the design point engine.

B. Develop constraint/thrust diagrams for the required flight maneuvers.

Use the computer program ACSYS to determine the solution space for the thrust and wing loading and to produce constraint/thrust diagrams. For engine retrofit the wing surface area, S, is fixed which allows lines of constant thrust to be plotted against the wing loading and thrust loading variables. When the aircraft performance requirements are plotted, the amount of sea-level thrust that the engine must produce in order to fly the maneuver can be determined.

C. Identify an appropriate mission to judge the fuel consumption improvements of each candidate engine.

The aircraft is "flown" using the mission analysis program MISS, to simulate the off-design characteristics of the engine and determine its fuel consumption. It

identifies shortcomings and whether or not the mission can be flown. MISS is a useful tool when testing candidate engines because it reports if the off-design solution fails to converge and why, as well as warn the user of an engine inlet which has to be resized to accommodate the incoming mass flow.

The mission analysis forms the basis for comparing the aircraft's overall fuel consumption while performing a baseline mission. The mission the aircraft will fly can either be modeled after its current mission profile or a new mission. If a change in the mission is required, then the aircraft can be flown in the new environment, but the overall engine characteristics may change considerably. The assumption was made that the aircraft's original mission profile was still valid for this investigation and that the engine was being redesigned to allow greater mission flexibility and fuel economy.

The output from MISS is used to compare weight fractions, overall fuel consumption, and whether or not the engine power is adequate for the aircraft without increased mass flow requirements. This last item refers to the fact that a baseline engine can be specified which produces much higher thrust than the one currently in use, but the space available in the airframe may not be large enough to accommodate it.

D. Generate solution surfaces and trend analyses for selecting candidate design point engines.

After selecting a design point Mach number and altitude, generate solution surfaces and perform trend analyses. This is accomplished using the ONX computer program to generate the design point data over ranges of compressor pressure ratio and bypass ratio. The solution surfaces and quality measures are then developed to relate the changes in F/m_0 and S at the design point.

By creating plots at different altitudes and Mach number, specific candidate engine designs can be selected for testing and to develop trends within the range of design variables.

E. Test the candidate engines using MISS.

A single point file is created using the ONX computer program for each candidate engine. This information is loaded into MISS along with the aircraft design point data. The engine's sea-level thrust and fuel consumption are obtained and the engine's position on the constraint/thrust diagrams determined. The fuel consumption is another indicator of the efficiency of the new design point and therefore an additional check on the improvement of one design over another.

F. Use linear regression to build the models.

The candidate design point engines tested represent only a small sample of all available solutions. In order to perform a thorough check of the solution surface all variations of design variables must be calculated in order to choose the best combination. A better method is to use linear regression analysis techniques to model the response.

Properly constructed models reduce the computer intensive work of running a mission analysis for each design point engine. Models are created which relate the design variables to the fuel consumption and sea-level thrust using the SAS software. From the models, and depending upon the number of variables, new engine designs are selected using any one or all of the following methods.

2 variables:

Generate surfaces which visually represent the effect of the two variables on the dependent variable and make selections.

Any number of variables:

a. Use SAS predicted values to rank the engines and make selections.
b. Identify the effect of each independent variable on the dependent variable and make selections.

The selected engines are then "flown" using MISS to see if the desired improvement was obtained. This in effect tests the validity of the model. If there is considerable error between the SAS predicted values and the MISS values, the

data base can be expanded with the new MISS observations until enough data points have been sampled to correctly characterize the response.

Through modeling, trends can be established and inferences made concerning the influence of one variable over another. This allows selection of a minimum number of design choices, since the trend established is based on the fit of the model to the data. This model only provides approximate solutions to any given set of design variables and care must be used in building the model as well as interpreting the results.

G. Produce the best candidate engines and plot their performance on constraint/thrust diagrams.

The constraint/thrust diagrams are used to present the changes in capability produced by the new engines while performing various maneuvers. The aircraft's takeoff performance, maneuverability, and max performance requirements can be evaluated with constraint/thrust diagrams. These diagrams provide a method for evaluating the merits of each design.

H. Select the new engine design point.

The final engine choice is based on the performance improvements indicated by the constraint/thrust comparisons and/or the ability to accomplish the mission with improved fuel economy. This engine is refered to as the baseline engine and is used to begin the design of the individual engine components.

This section has discussed the analysis tools used and the design scheme developed for jet engine retrofit. The next section will consider two applications of the process to demonstrate its utility. The process described above is outlined in the flow chart shown in Figure 16.

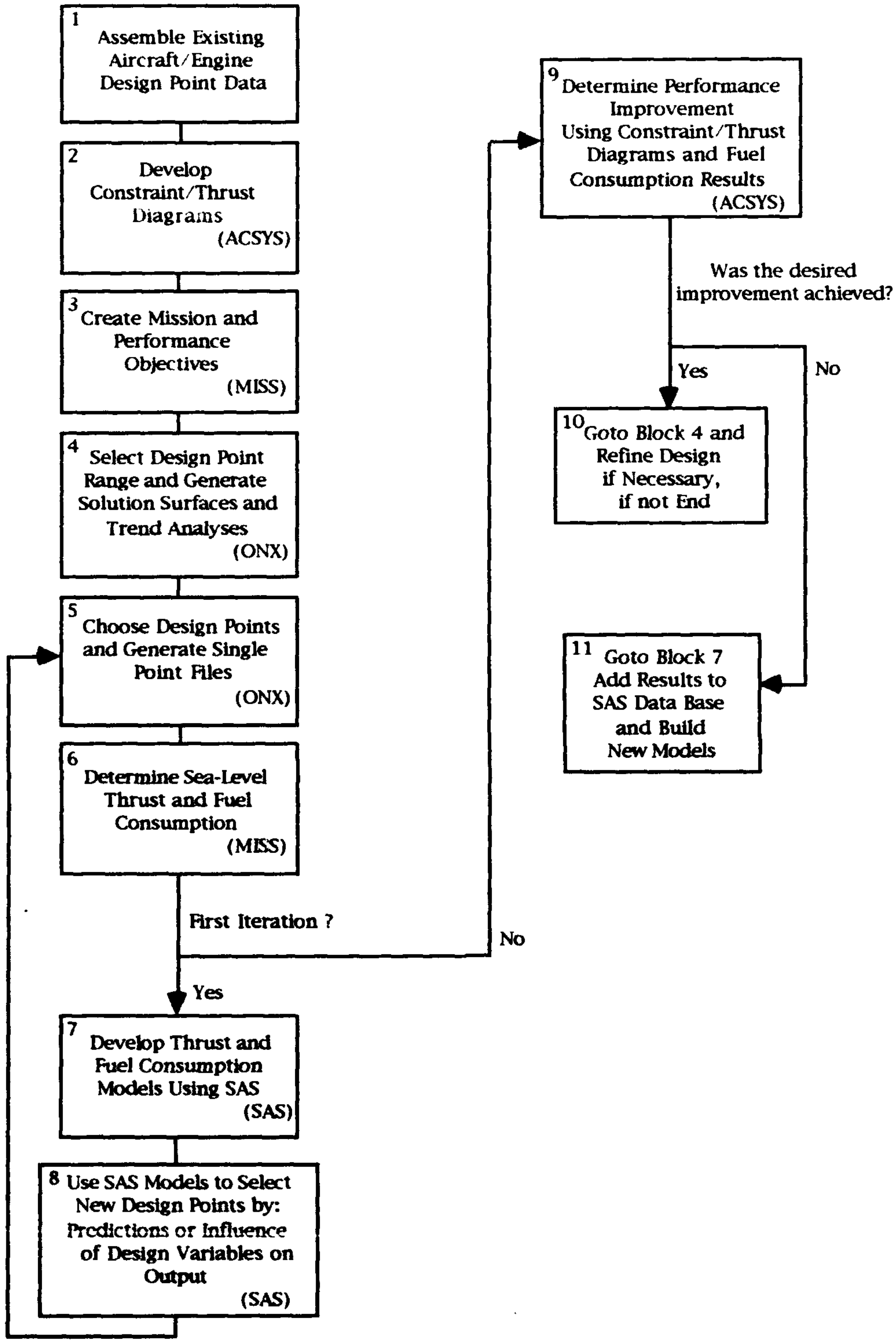


Figure 16. Solution Scheme Flow Chart

III. Example Analysis

The two engine retrofit examples presented in this section, used the Air-to-Air Fighter, AAF, developed in Reference 6 Chapters 1 through 6. This particular engine, airframe, and mission combination was selected in order to provide a reference against which to make comparisons and point out the performance improvements, compromises and disappointments.

The new engine may be designed to perform any of a number of parts of the mission better than its predecessor. Since the engine retrofit problem is based on a high performance fighter aircraft, the goal of these design examples was to identify engines which will increase the sea-level thrust and decrease the fuel consumption. If an engine can fulfill both of these requirements while simultaneously satisfying any other new mission requirement, then it is a standout candidate for retrofit.

The first example demonstrates the use of the design scheme when examining the impact of two variables on the engine design. This analysis would be most useful when searching a small region for optimizing the final baseline engine. This search focused on the compressor pressure ratio and the bypass ratio.

The second example considers applying the design scheme to a multivariable problem. This method is most useful when the search is to be conducted over a wide range of design variables and the intent is to rapidly reduce the limit of possible choices. This example considered variations in Mach number, altitude, compressor pressure ratio, bypass ratio, fan pressure ratio, and mass flow rate..

Preliminary information for the examples is developed in sections 3.1 through 3.3. Section 3.4 contains the analysis of the two-variable design approach and section 3.5 contains the six-variable design approach. The engines developed by the two approaches are then compared in Section 3.6.

3.1. Design Point Airframe and Engine

The search for the new engine began after extracting the appropriate data from the existing AAF design developed in Chapters 1 through 6 of Reference 6. This information consists of the mission and performance specification for the AAF along with the airframe and baseline engine design parameters. The information extracted from the design is outlined below:

A. Aircraft Performance and General Information

The existing performance requirements and mission for the AAF were used to develop the constraint diagrams, constraint/thrust diagrams, and mission analysis. The performance requirements are:

Takeoff distance	1500 ft
Landing distance	1500 ft
Max Mach number at maneuver weight	M=2.0/40,000 ft
Supercruise requirement at maneuver weight	M=1.5/30,000 ft
Acceleration at maneuver weight	M=0.8 to 1.6/30,000 ft in $t \leq 50$ s
Sustained g level at maneuver weight	$n \geq 5$ at M=0.9/30,000 ft
	$n \geq 5$ at M=1.6/30,000 ft

The maneuver weight of this aircraft was defined as the aircraft's weight when carrying two AIM-9L missiles, 250 rounds of ammo, and 50% of internal fuel. The weight of the armament and stores is:

Armament/Stores	
AIM-9L Sidewinder Missile	191 lb
AMRAAM	326 lb
25mm Cannon	
Cannon weight	270 lb
Rate of fire	3600 rpm
Ammunition feed system weight (500 rounds)	405 lbs
Ammunition (25mm)	
Weight (fired rounds)	550 lb
Casings weight (returned)	198 lb

The aircraft design weight of 24,400 lbs was used with a wing loading of 64 and thrust loading of 1.2. These values were used for the mission analysis phase of the engine design.

The aircraft weight fractions were kept the same as those used in the original analysis for the AAF. These were 1.0 for takeoff, 0.58 for landing, and 0.78 for maneuvers.

For the aircraft performance analysis, the engine thrust model and TSFC model for the low bypass ratio mixed turbofan of the ACSYS program are assumed adequate to model the engine for this part of the development.

The aircraft drag model used was the default future fighter model in the programs ACSYS and MISS. Even though a user input model could have been used, it was considered unnecessary since the intent is to provide insight into the effect of various engine designs on performance. As long as there is consistent use of a particular drag model, the analysis is considered valid.

The following is a list of additional design information for the aircraft used with the computer programs throughout this analysis:

Takeoff	Landing	Miscellaneous
k_{TO}=1.2	k_{TD}=1.15	C_{Lmax}=2.0
μ_{TO}=0.05	μ_B=0.18	n=5
t_R=3 s	C_{DRc}=0.5348	q_{max}=2133 lb/ft^2
	t_{FR}=3 s	

This summarizes the information used from the existing airframe design and performance requirements.

B. Baseline Engine Information

The baseline engine data for the AAF engine developed in Reference 6 is presented below and will be used as the starting point for the design search.

Table 2. Baseline AAF Engine Design Point Data (6:207)

M_0=1.6	M_5=0.4	C_{TO}=0.016
h=35,000 ft	T_{t4}=3200 R	A_1=3.287 ft^2
π_c=20	T_{t7}=3600 R	A_{10}=4.028 ft^2
π_c'=3.7	m_0=94.5 lbm/s	L=4.087 ft
α=0.55		

The engine diameters and nozzle length will be held constant to insure a fit in the airframe. The maximum operating limits on T_{t4} and T_{t7} were fixed at 3200 R and 3700 R respectively, to minimize the number of variables and to identify a design point which does not require increased temperatures to improve performance. Any increase in turbine inlet temperature leads to an increase in fuel consumption and no benefit is achieved. (6.133)

Since the idea behind the retrofit is to find a comparable engine which will perform adequately, the search was limited to engines with the same design performance parameters as the AAF. This allowed direct comparison of the proposed engines to the existing AAF design without having to account for the influence of the component efficiencies or any of the other values. The engine design performance parameters used are presented in Table 3.

Table 3. Component Design Performance Parameters (6:125)

Description	Design Value
Polytropic Efficiency	
Fan (e_c')	0.89
High Pressure Compressor (e_c)	0.90
High Pressure Turbine (e_{th})	0.89
Low Pressure Turbine (e_{tl})	0.91
Total Pressure Ratios	
Inlet (π_{dmax})	0.97
Burner (π_b)	0.97
Mixer (π_{Mmax})	0.97
Afterburner (π_{AB})	0.96
Nozzle (π_n)	0.98
Component Efficiency	
Burner (η_b)	0.98
Afterburner (η_{AB})	0.97
Mechanical	
Low Pressure Spool (η_{mL})	0.99
High Pressure Spool (η_{mH})	0.98
Power Takeoff (η_{mP})	0.98
Fuel Heating Value (h_{PR})	18,000 BTU/lbm
Afterburner Total Temperature (T_{tmax})	3600 R
Turbine Cooling Air	
T_{t4}>2400 R	$\varepsilon_1=\varepsilon_2=(T_{t4}-2400)/16{,}000$
$T_{t4}\leq$2400 R	$\varepsilon_1=\varepsilon_2=0$

Required inputs to MISS are the design values for the conditions at engine station #3. The values for π_{cmax}, total pressure, P_{t3}, total temperature, T_{t3}, and the low and high pressure spool rpm are determined by knowledge of the design considerations behind each term.

The total temperature at station 3 is dictated by the maximum design temperature allowable based on the material properties at this location. A rule of thumb is that the compressor discharge temperature upper limit is approximately 1660 R (6:288). The engines in a future fighter should be able to exceed this temperature and so a reasonable value for the new engine was selected at a temperature of 1760 R.

The total pressure at station 3, is a function of both the maximum pressure which the burner can accept, for structural design reasons and to maintain combustion stability, as well as the maximum pressure which can be successfully raised by the compressor. The value of 350 psia was selected based on comparison with previous engines using similar design temperatures and should provide a reasonable estimate for the AAF (6:303).

The max spool rpm is based on the efficiency of the compressor and turbine as a function of engine speed. The efficiency of turbomachinery is typically nearly constant between 70% and 100% of engine speed rpm. The efficiency tends to drop off steadily at values above 110% and below 60%. For the maximum allowable rpm select 110% to allow for some migration during the high speed maneuvers away from the 100% limit. The assumption of constant efficiency would not apply to regions of the flight envelope requiring high altitude and low Mach number or low altitude and high Mach number flight (6:166).

Setting a maximum design value for π_c forces the engine to lower T_{t4} to keep this limit from being exceeded. The result is a decrease in available thrust during any flight condition for which the limiting value of π_c would be exceeded. The effect of this limitation is illustrated by the OFFX computer program output for three calculations presented below.

Setting	F (lbf)	Limitation	T_{t4} (R)	π_c
P_{t3}=300 psia	9458	LP spool rpm	2888.7	26.35
π_c=26.35	9459	LP spool rpm	2888.8	
π_c=20	7329	π_c max	2580.8	

The first calculation used a value for P_{t3} of 300 psia and the limit for the calculation was the low pressure, LP, spool rpm. The value for the compressor pressure ratio was 26.35 with an uninstalled thrust of 9458 lbf and a turbine inlet temperature of 2888.7 R. When the limit on P_{t3} was removed and a limit of 26.35 placed on π_c, the results were the same, and the limiting factor was the LP spool rpm. When π_c was reduced to 20 the engine required a reduction in T_{t4} in order to keep π_c from being exceeded. The result of this reduction was an over 2000 lbf reduction in available thrust. Lowering the ratio of turbine inlet temperature to compressor inlet temperature, i.e. decreasing T_{t4}, reduces the compressor flow rate, speed, and pressure ratio resulting in a reduction of available thrust and engine efficiency (11:50). The larger the engine temperature ratio, the higher the engine efficiency and specific excess power.

The final selection of limits for the design values at station 3 are:

π_c=none
P_{t3}=350 psia
T_{t3}=1760 R
LP Spool RPM=110 %
HP Spool RPM=110 %

The engine design values described in this section were used for the remainder of the analysis.

3.2. Constraint Diagrams

The constraint diagram and constraint/thrust diagrams for the AAF were constructed to determine the approximate thrust levels required for the new engine design. The constraint information used with ACSYS to generate the plots are presented in Table 4.

The constraint diagram produced with this data is presented as Figure 17. The design point for the AAF aircraft developed in Reference 6 is noted on the diagram and the solution space presented. The usefulness of this diagram was enhanced by adding a line of constant sea-level thrust of 30,000 lbf as shown in Figure 18. The required T_{SL} is therefore a little over 30,000 lbf to power the aircraft through the maneuvers at the weight fractions selected.

Table 4. Constraint Analysis Information

Takeoff (No Obstacle)

- $\beta=1.0$
- Altitude=2000 ft
- $C_{DR}=0.1$
- Mach # during roll=0.1
- $C_{Lmax}=2.0$
- $k_{TO}=1.2$
- $\mu_{TO}=0.05$
- $t_R=3.0$ s
- $S_{TO}=1500$ ft
- Afterburner=ON

Landing (No Obstacle)

- $\beta=0.56$
- Altitude=2000 ft
- $C_{DR}=.5348$
- Fraction of C_{Lmax} for braking=0.61
- $C_{Lmax}=2.0$
- $k_{TD}=1.15$
- $\mu_{TD}=0.18$
- $t_{FR}=3.0$ s
- $S_{LD}=1500$ ft
- Drag Chute

Constant Altitude/Speed Cruise

- $\beta=0.78$
- M=1.5
- Altitude=30,000 ft
- Afterburner=OFF

Constant Altitude/Speed Turn

- $\beta=0.78$
- M=0.9
- Altitude=30,000 ft
- 5g's
- Afterburner=ON

Constant Altitude/Speed Cruise

- $\beta=0.78$
- M=2.0
- Altitude=40,000 ft
- Afterburner=ON

Constant Altitude/Speed Turn

- $\beta=0.78$
- M=1.6
- Altitude=30,000 ft
- 5g's
- Afterburner=ON

Horizontal Acceleration

- $\beta=0.78$
- Altitude=30,000 ft
- dV/dt=15.9
- Afterburner=ON

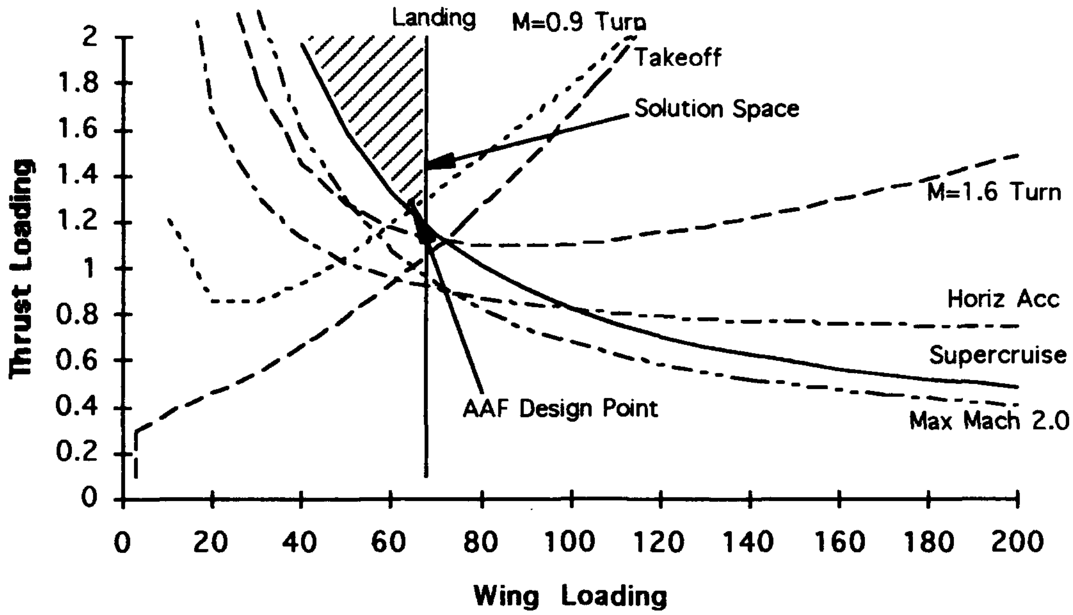


Figure 17. AAF Constraint Diagram

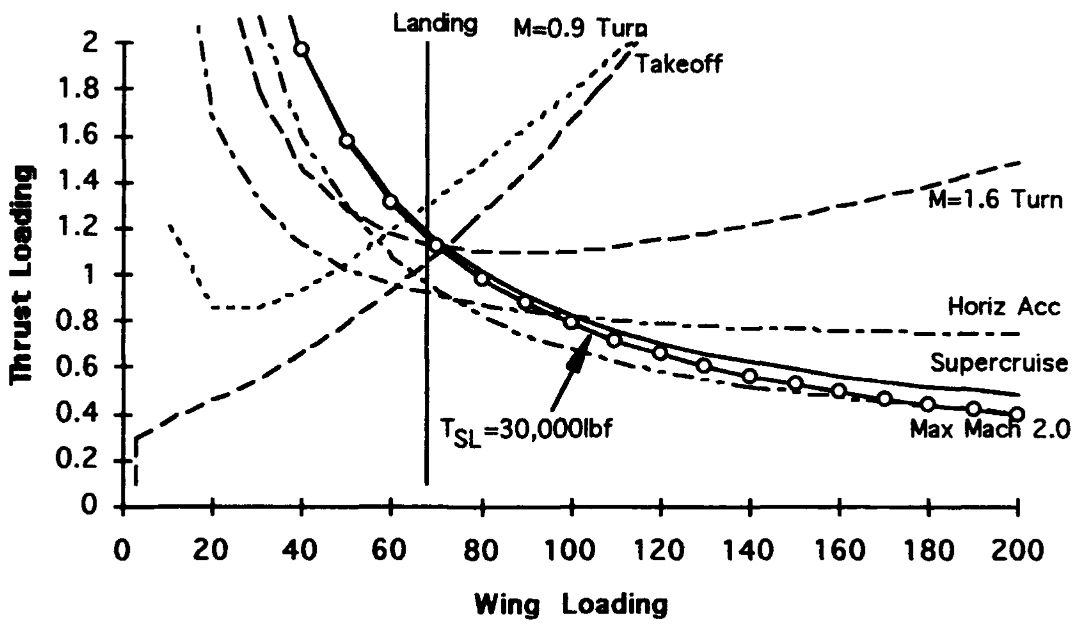


Figure 18. AAF Constraint Diagram with Constant Thrust Line

Constraint/thrust diagrams were constructed for the two maneuvers which border the solution space as shown in Figure 17 and the M=1.6 turn. The M=1.6 turn, M=0.9 turn and supercruise diagrams are shown in Figures 19 through 21. An additional constraint/thrust diagram, Figure 22, was developed to show, that for this particular aircraft , the subsonic cruise, M=0.9, leg will not have an impact on the solution for any pair of engines which produce more than 20,000 lbf (total) of thrust, since the constraint lines lie well below the lowest thrust line (for the wing loadings being considered).

Note from the constraint/thrust diagrams, Figures 19 through 22, that at a wing loading of 64 and a maneuvering β of 0.78, the engines will have to produce a minimum T_{SL} of 30,000 lbf (total) in order to perform as required. This information will be used later in selecting the best choice.

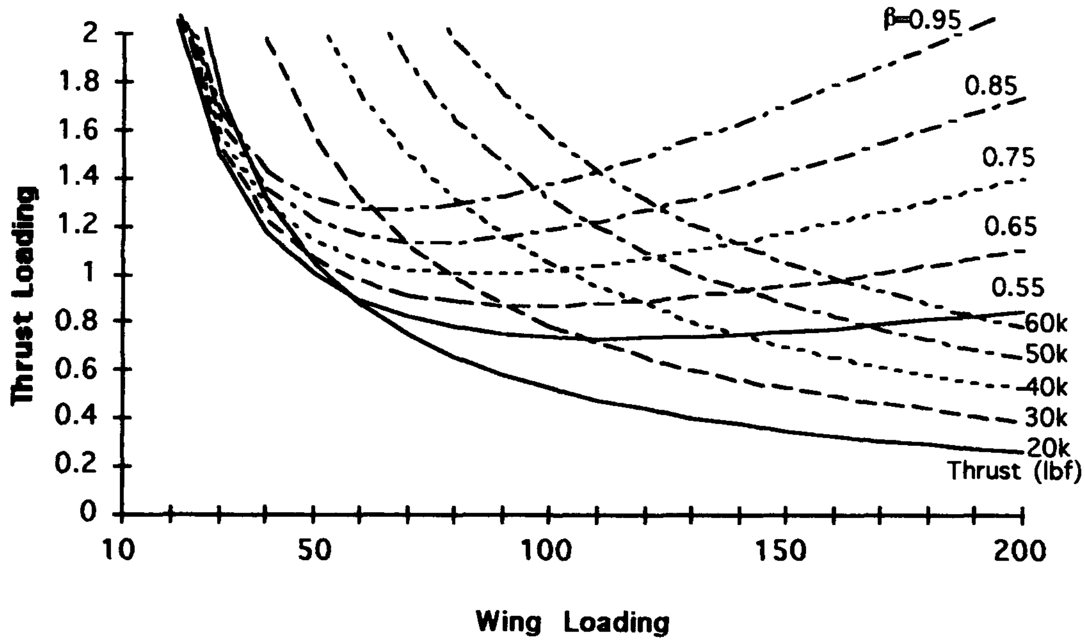


Figure 19. Constraint/Thrust Diagram, M=1.6, 30k ft, 5g Turn

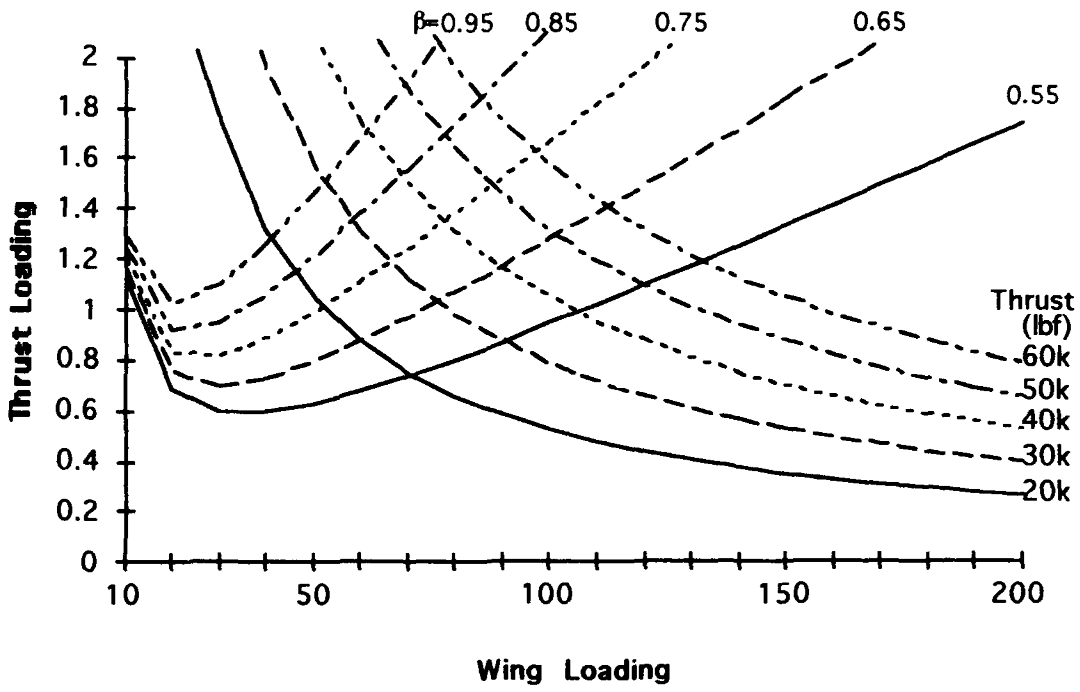


Figure 20. Constraint/Thrust Diagram, M=0.9, 30k ft, 5g Turn

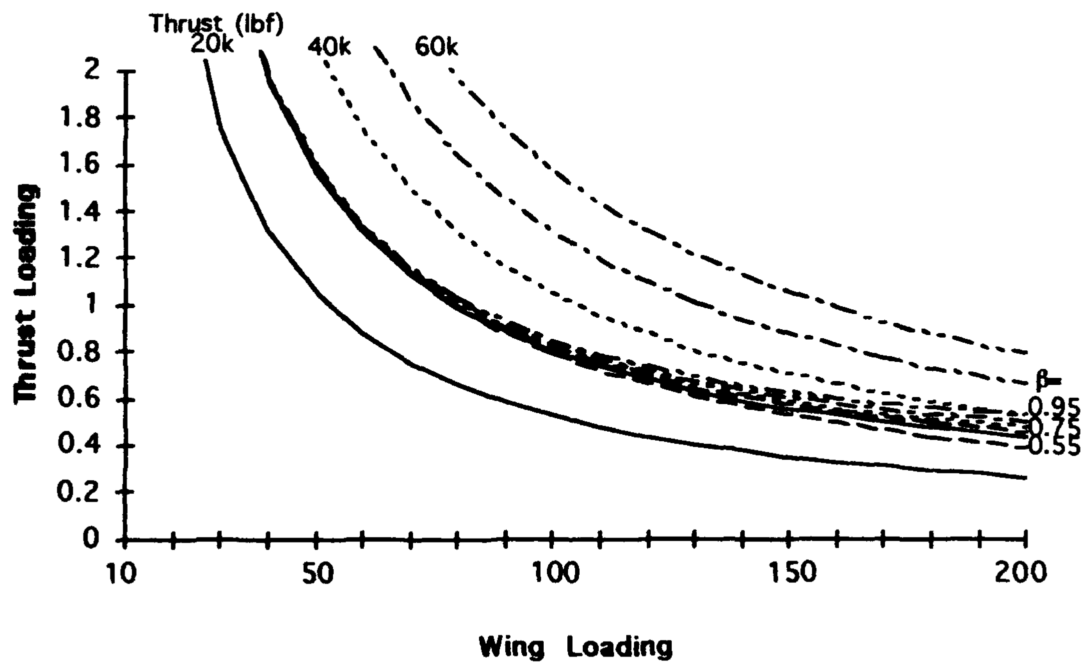


Figure 21. Constraint/Thrust Diagram, M=1.5 Cruise, 30k ft

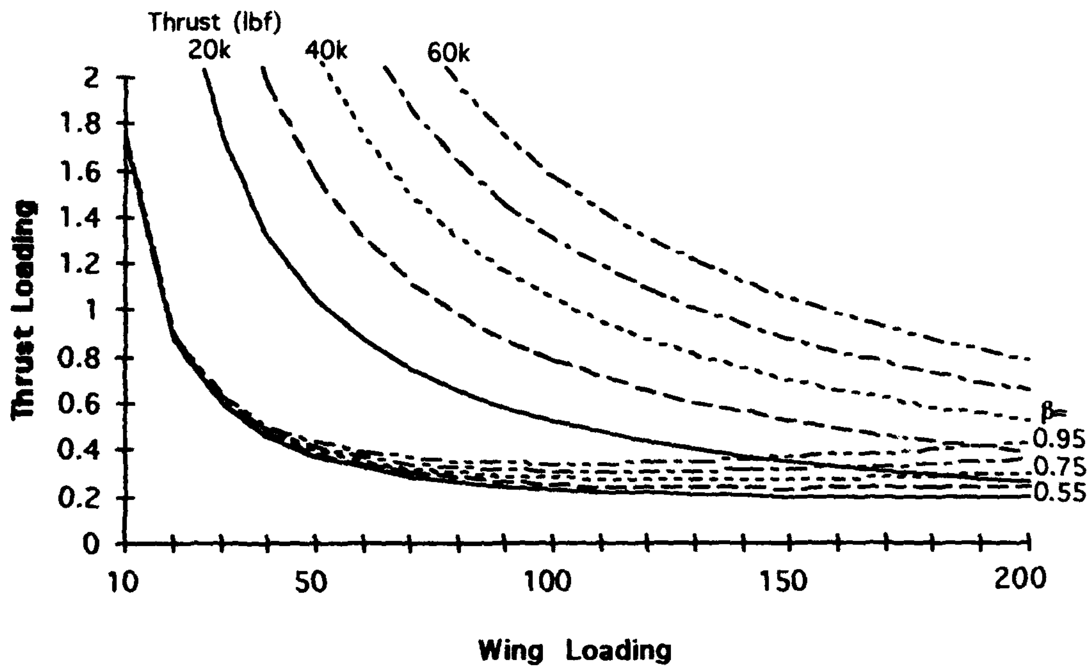


Figure 22. Constraint/Thrust Diagram, M=0.9 Cruise, 30k ft

3.3. Mission Development

The aircraft was "flown" using the computer program MISS to simulate the mission. MISS performs an off-design analysis to determine whether or not the engine has acceptable off-design performance characteristics while calculating the fuel consumption over the length of the mission. This analysis will be used when the new engine design points are selected and is presented here as a preliminary step in preparing for evaluating the design point engines.

The mission particulars were taken directly from the AAF request for proposal presented in Chapter 1 of Reference 6. The following mission legs were used:

1. Warmup
2. Takeoff Acceleration
3. Takeoff Rotation
4. Horizontal Acc
5. Climb and Acc Legs (4)
6. Constant Alt/Speed Cruise
7. Loiter
8. Horizontal Acc
9. Constant Alt/Speed Cruise
10. Constant Alt/Speed Turns (2)
11. Horizontal Acc
12. Deliver Expendables
13. Constant Alt/Speed Cruise
14. Constant Speed Climb Legs (2)
15. Constant Alt/Speed Cruise
16. Loiter

The mission leg data and airframe data loaded into the program are presented for each leg in Appendix D. All candidate engines can now be input to the program MISS using an on-design single point reference file from ONX as described in Reference 9.

The mission outlined above was used for determining the fuel consumption of each engine for each leg and for the complete mission. The sea-level thrust was determined by performing a single calculation at max power using the warm-up leg.

3.4. Two-Variable Analysis

3.4.1. Generation of Solution Surfaces

The preliminary information for "fly.ng" and comparing the engines has been assembled and the search for possible new engines can begin. This section covers the construction of the solution surfaces for the two-variable problem.

The design point of the existing AAF engine was presented in Table 2 and was selected as the best point to begin the design search in order to identify if any improvements could be made at this design condition.

Using the ONX computer program, multiple calculations for uninstalled specific thrust, F/m_0, and uninstalled specific fuel consumption, S, were made by varying π_c from 10 to 30 while α varied from 0.4 to 0.65; the results are presented in Appendix E. The solution surfaces were generated by eliminating those combinations of π_c and α which failed to meet the criteria that M_5 and M_5' lie between 0.4 to 0.6. This generated two plots, one of F/m_0, and another of S, see Figures 23 and 24.

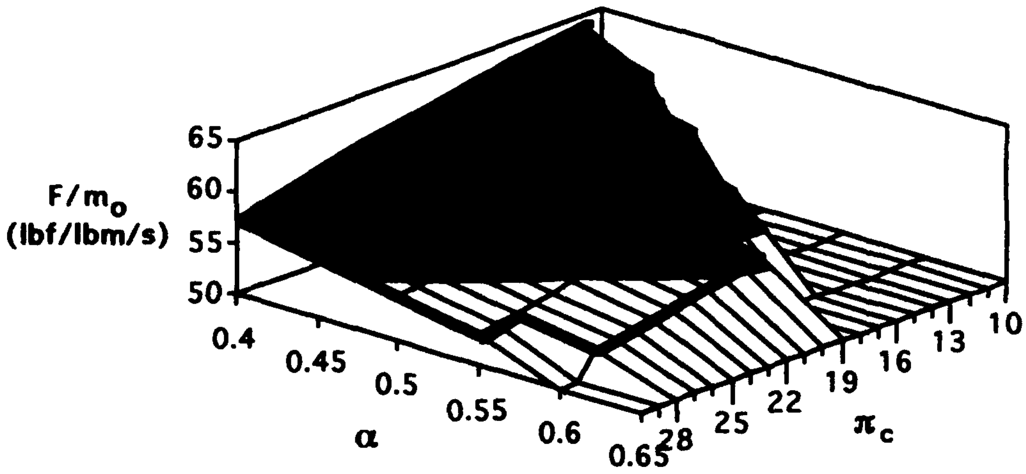


Figure 23. F/m_0 Solution Surface, M=1.6, 35k ft

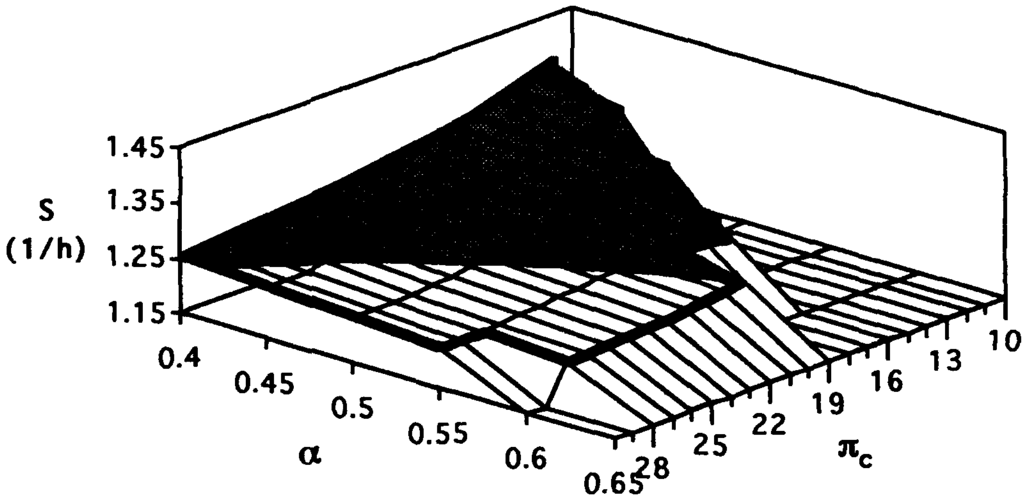


Figure 24. S Solution Surface, M=1.6, 35k ft

The heavy black border outlines the solution surface and represents the best set of possible solutions. The shading corresponds to the areas which lie between the plotted values on the dependent variable axis.

The solution surface is used to provide a visual indication of the available solution space after all restrictions have been imposed (both surfaces represent the same set of solutions but they are plotted against a different variable). The search is restricted to those combinations of π_c and α within the bordered region since searching in the lower regions outside the border will likely result in a poor engine choice.

Once the solution surfaces were plotted, the next step was to determine the values of the quality measures associated with the surfaces.

3.4.2. Generation of ONX Trend Data

The solution surface data was collected for the ranges of π_c and α, see Appendix E, and from this information the trend analysis plots were constructed. These plots provide insight into the tradeoffs between F/m_o and S.

The AAF engine plots for the quality measures Q_1, Q_2, and Q_3 are shown in Figures 25 and 26. Figure 25 indicates that as the compressor pressure ratio and bypass ratio are increased, there is a continual decrease in both F/m_0 and S. F/m_0 tends toward a constant decrease of between 0.006 and 0.007 for every unit increase in π_c while S continues to decrease but at a slower rate. For a fuel efficient engine the tendency would be to move toward increasing values of π_c, but for every decrease in fuel consumption of -0.002 to -0.003, there is a correspondingly large decrease in thrust between -0.006 and -0.007. This plot shows the impact of each increase in π_c or change in α.

Figure 26 was used as another gage of the rate at which the change in π_c would effect the changes in F/m_0 and S. The decrease in thrust is much larger than the decrease in fuel consumption at the higher compressor pressure ratios. The thrust is decreasing twice as fast as the decrease in fuel consumption for π_c above 22. At Q_3=1, both quantities are changing at the same rate.

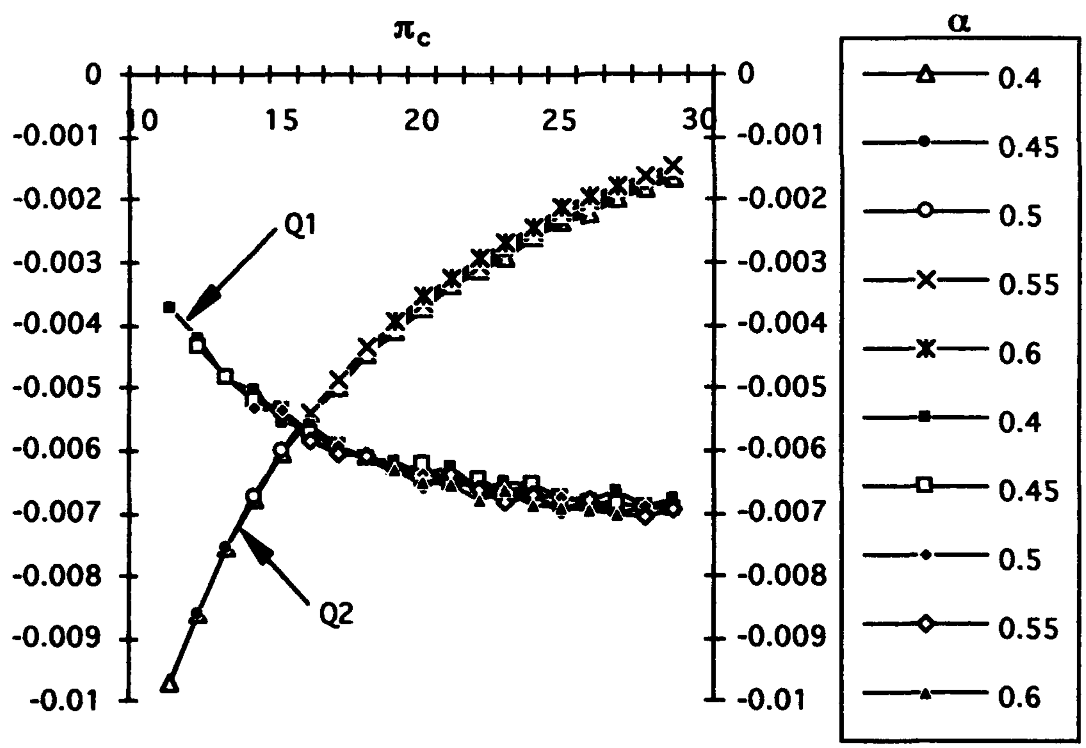


Figure 25. Quality Measures Q_1 and Q_2 for M=1.6, 35k ft

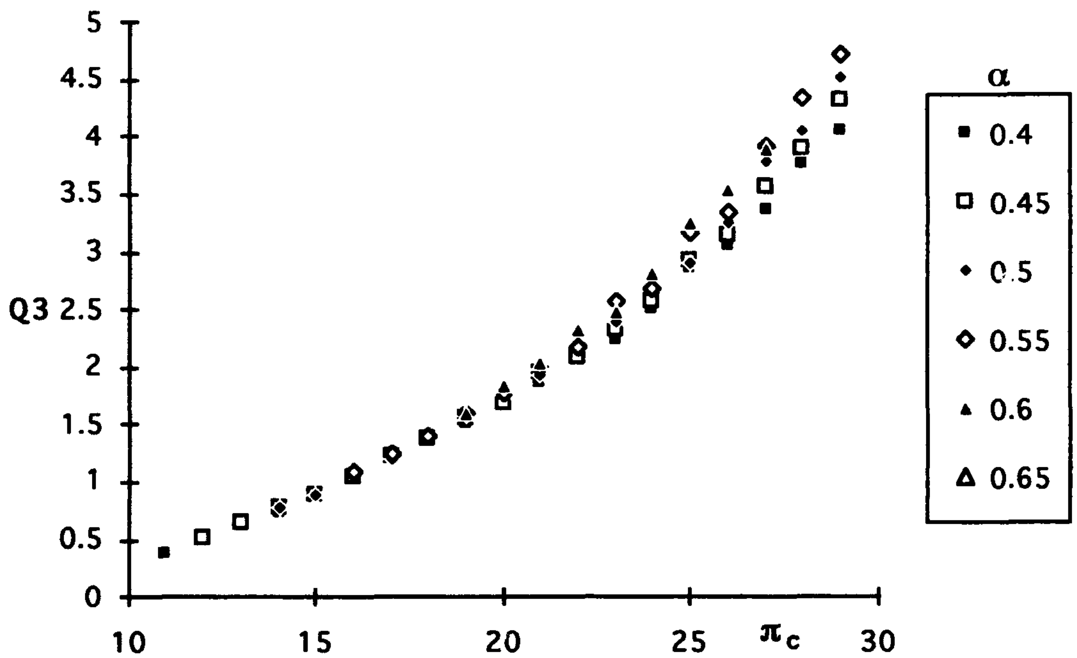


Figure 26. Quality Measure Q_3 for M=1.6, 35k ft

Selecting the appropriate design point ranges required careful examination of the solution surface and the trend data above. It can be seen that when comparing any two bypass ratios at the same compressor pressure ratio there is little change in the measures Q_1 or Q_2. This indicates that $F/\dot{m}_0$ and S are dependent upon π_c rather than α.

Toward compressor pressure ratios to the left of the intersection of Figure 25, the fuel consumption is changing rapidly while the thrust is changing at a slower rate. Engines in this region will produce higher thrust levels with higher fuel consumption. This engine choice would suit a high performance requirement. Engines to the right of the intersection begin to exhibit increased fuel economy but at a loss of thrust. An engine at the far right would only be slightly more fuel efficient than those in the middle since fuel consumption is not changing very rapidly. The best choice for improving overall performance of the AAF is to select a range where the change in thrust is still small while the maximum change in fuel consumption has dropped to a reasonable value. This tradeoff occurs to the right of the intersection.

The quality measure Q_3, Figure 26, was used for limiting the range of design variables to those where the change in thrust is no more than twice the change in fuel consumption. This was an arbitrary choice made for this investigation and, when combined with the observations made from Figure 25, results in limiting possible engine combinations to those between compressor pressure ratios of 15 and 22 with bypass ratios of 0.4 to 0.6. Also, the selected range for the design variables falls on the solution surfaces of Figures 23 and 24 where off-design solutions should be plentiful.

Using the solution surfaces to guide the choice of design points, a sampling was made which would characterize the engines performance over this region. The selected combinations of π_c and α were:

π_c	α
15	0.4
16	0.5
18	0.45
18	0.55
20	0.5
20	0.6
22	0.55

The selection so far has centered around minimizing S while maximizing F/m_0. The solution surface has been generated to locate the possible solution areas and the quality measures have been used to examine the relative changes between the thrust and fuel consumption.

The selected engines were "flown" using MISS to determine their mission fuel consumption and sea-level thrust. The mass flow rate had to be increased from the AAF baseline engine design point value of 94.5 lbm/s to 100 lbm/s in order to meet all performance requirements. This will become the standard mass flow rate used in this analysis.

The next step was to build a model to characterize all possible combinations of π_c and α over the selected ranges.

3.4.3. Modeling Analysis

After the candidate design point engines were selected they were "flown" to see if they perform the off-design mission. All candidates performed well and their max power sea-level thrust, T_{SL}, and mission fuel consumption were determined. The results of operating the selected engines is shown below:

π_c	α	T_{SL}	Fuel Consumption
15	0.4	14,539	8579
16	0.5	14,400	8410
18	0.45	14,465	8314
18	0.55	14,375	8268
20	0.5	14,394	8243
20	0.6	14,166	8159
22	0.55	13,079	8143

The change in T_{SL} and fuel consumption over the range selected appears to indicate that the best choice is somewhere in the middle. The options at this point are either to run all the design combinations around the middle values of π_c=18 and α=0.5 or use modeling to help select the next best choice.

The results above were used with the SAS modeling software to develop 6 models, 3 for thrust and 3 for fuel consumption, by using various combinations of the independent variables π_c and α, see Appendix F. The equations for the thrust and fuel consumption models determined by SAS are presented below.

$$Thrust_1 = 498.7\pi_c + 22357\alpha - 67.77\pi_c^2\alpha^2$$
$$Thrust_2 = 2221.18\pi_c - 17745\alpha - 75.6\pi_c^2 + 882.24\pi_c\alpha$$
$$Thrust_3 = 2266.2\pi_c - 19381\alpha - 64.63\pi_c^2 + 17425\alpha^2$$
$$Fuel_4 = 11472 - 235.76\pi_c - 1749.84\alpha + 4.39\pi_c^2 + 58.1\pi_c\alpha$$
$$Fuel_5 = 11496 - 236.1\pi_c - 1833\alpha + 5.2\pi_c^2 + 1126.12\alpha^2$$
$$Fuel_6 = 17739 - 758.24\pi_c - 17795\alpha + 6.45\pi_c^2 + 1487.02\pi_c\alpha - 28.95\pi_c^2\alpha^2$$

The thrust models were produced without an intercept since this resulted in the best fit of the data, see Appendix C. The statistics used to evaluate the fit of each model and the influence of the variables are presented below.

Model 1 ($Thrust_1$)	F Value=2034.14	Prob>F=0.0001
	Variables	Prob>|T|
	π_C	0.0087
	α	0.004
	$\pi_C^2\alpha^2$	0.0005
	R-Square	0.9993
Model 2 ($Thrust_2$)	F Value=8470.513	Prob>F=0.0001
	Variables	Prob>|T|
	π_C	0.0161
	α	0.3635
	π_C^2	0.0509
	$\pi_C\alpha$	0.3938
	R-Square	0.9999
Model 3 ($Thrust_3$)	F Value=8273.967	Prob>F=0.0001
	Variables	Prob>|T|
	π_C	0.0224
	α	0.3861
	α^2	0.4142
	π_C^2	0.0177
	R-Square	0.9999
Model 4 ($Fuel_4$)	F Value=61.723	Prob>F=0.016
	Variables	Prob>|T|
	Intercept	0.0038
	π_C	0.0925
	α	0.2703
	π_C^2	0.2703
	$\pi_C\alpha$	0.6568
	R-Square	0.9920
Model 5 ($Fuel_5$)	F Value=60.128	Prob>F=0.0164
	Variables	Prob>|T|
	Intercept	0.0043
	π_C	0.0957
	α	0.543
	α^2	0.6924
	π_C^2	0.1301
	R-Square	0.9918
Model 6 ($Fuel_6$)	F Value=89.844	Prob>F=.0799
	Variables	Prob>|T|
	Intercept	0.1381
	π_C	0.2599
	α	0.3269
	π_C^2	0.2362
	$\pi_C\alpha$	0.3424
	$\pi_C^2\alpha^2$	0.3524
	R-Square	0.9978

To evaluate the fit of the data and select the best models, the F Value, Prob>F, and the R-Square statistics were used. Large F Values indicate that the mean of the data did not happen by chance while Prob>F values close to 0.0001 indicate that the model explains a significant portion of the variation in the data. The R-Square value measures the fraction of the total variation due to the variables and the closer this value is to 1.0 the better the model is at accounting for variation in the data.

The Prob>F statistic for models 1 through 3 indicate that all models explain a significant portion of the variation in the data. Therefore, the best models were selected based on the largest F Values and the R-Square values which were closest to 1.0. This resulted in selecting thrust models 2 and 3 as the best fit of the data.

For models 4 through 6,the Prob>F values vary from 0.016 to 0.0799 and indicate that models 4 and 5 appear to be the best fit of the data. The F Value, on the other hand, indicates that models 4 and 6 are the best. The final choice was made by comparing the R-Square values which are closest to 1.0 for models 4 and 6. The best fit to the data for the thrust and fuel consumption was therefore achieved with models 2, 3, 4 and 6.

The fit statistics also indicate the influence of each variable on the output. This is determined by the Prob>|T| value which tests to see if the slope of the data is zero for that variable. The closer this measure is to 1.0 indicates that the variable has very little influence on the output.

The fit statistics show that π_c has a stronger influence on the output of most models, since the Prob>|T| is lower for the π_c terms than the α terms. For the fuel consumption models, the Prob>|T| indicated the strongest influence for all the models was due to the intercept. This is to be expected since the intercept value is derived from the lack of fit of the model and is tailored to account for any residual error. This results in the strong influence.

The best way to show the influence of the two variables is to plot each model over the range of π_c and α values under consideration. The equations for models 2, 3, 4 and 6 were used to generate surfaces over the range of π_c from 15 to 22 and α from 0.4 to 0.6. These surfaces are presented in Figures 27 through 30.

Figure 27 presents the modeled sea-level thrust determined by the equation $Thrust_2$. It indicates that the maximum thrust is achieved at compressor pressure ratios around 18 and bypass ratios of 0.4 while the minimum thrust occurs at the compressor pressure ratio of 22 and bypass ratio of 0.4. Figure 28, model $Thrust_3$, shows a similar trend but indicates less influence by the bypass ratio as indicated by the small slope along lines of constant π_c. These two surfaces provide insight into the engine's sea-level off-design performance at various combinations of π_c and α.

Figure 29 presents the modeled fuel consumption determined by equation $Fuel_4$. It indicates that the maximum fuel consumption will occur at a compressor pressure ratio of 15 and bypass ratio of 0.4 while the minimum fuel consumption occurs at a compressor pressure ratio of 22 and bypass ratio of 0.6. Figure 30, model $Fuel_6$, indicates less influence on the fuel consumption by the bypass ratio but still follows the same general trend as Figure 29. These two surfaces show how the mission fuel consumption varies with various combinations of π_c and α.

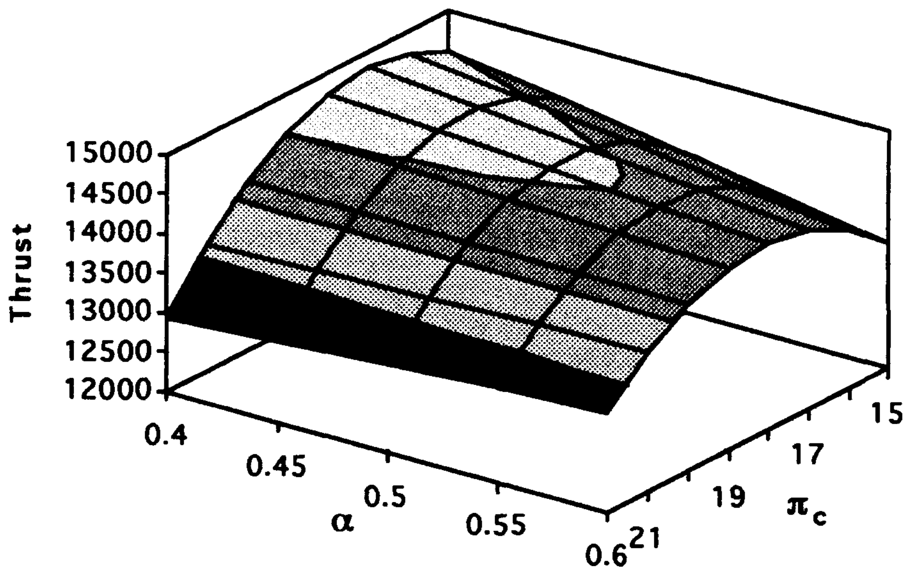


Figure 27. Model #2 Surface

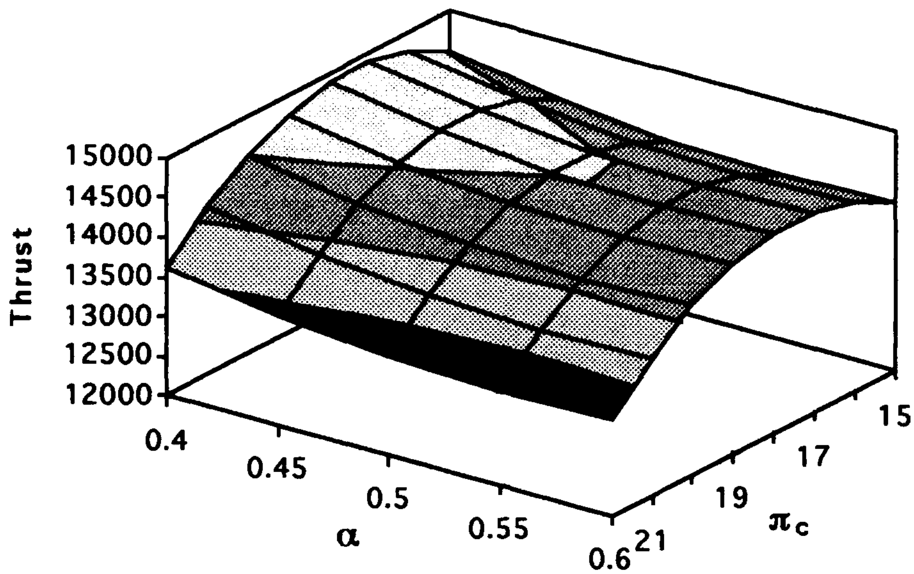


Figure 28. Model #3 Surface

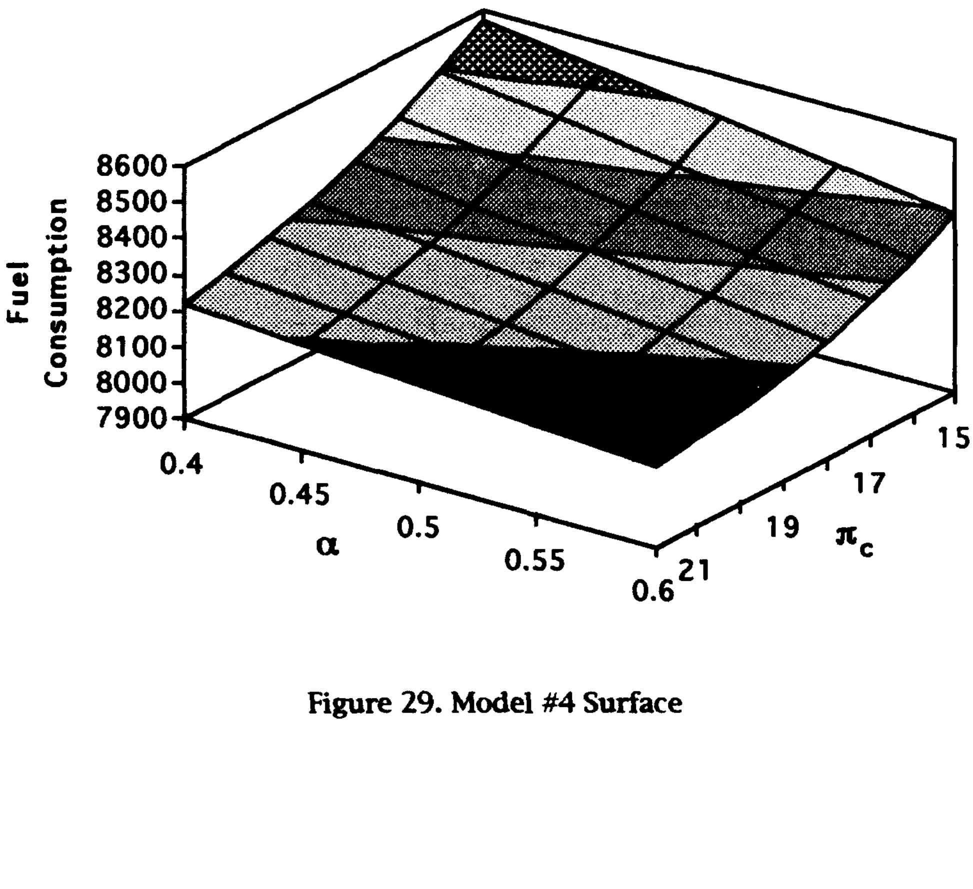


Figure 29. Model #4 Surface

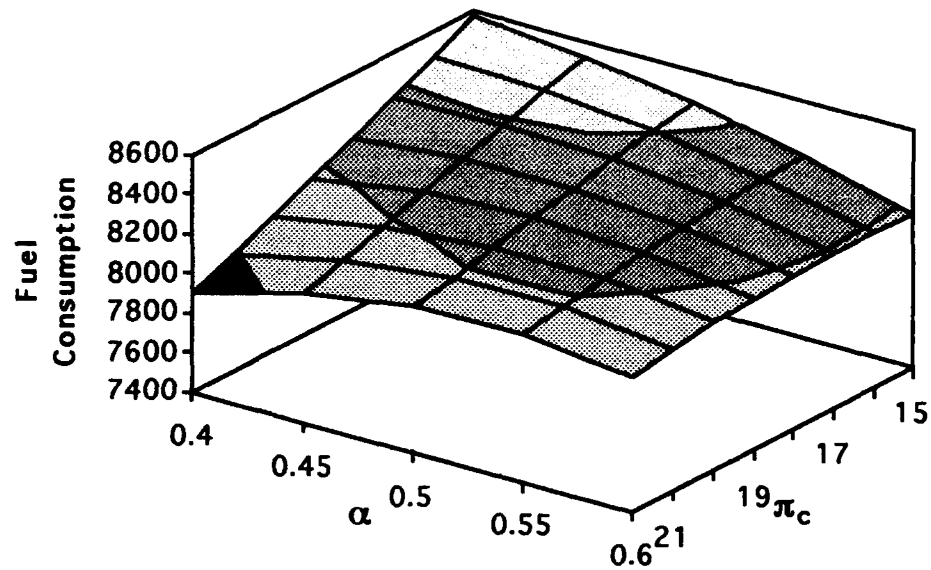


Figure 30. Model #6 Surface

After viewing these surfaces, the thrust levels tend to be highest at a π_c of about 18 and tend to level off at α about 0.5. The fuel consumption is high in this region but tends to decrease at high π_c and high α values for model 4 while decreasing at high π_c and low α values for model 6. Since the trends in thrust and fuel consumption are not complementary, the choice was made to continue to search for a design point engine at π_c values between 18 to 20 and α between 0.5 and 0.6.

Now that the models have been developed, the search is trivial and was accomplished using the SAS program rather than running another set of ONX single point files and MISS analyses. The SAS program took about 4 minutes to run a check on a wide range of possible engines in the new design range and determine what the predicted values would be. Twenty three combinations of π_c and α were used with SAS to calculate the fuel consumption and sea-level thrust. The following engines are numbered starting at 8 since the first seven engines form the initial data base. The SAS results are presented in Appendix F.

Number	π_c	α	Number	π_c	α
8	22	0.65	16	19	0.6
9	22	0.45	17	19	0.45
10	21	0.6	18	17.5	0.5
11	21	0.55	19	17	0.6
12	21	0.4	20	17	0.4
13	20	0.65	21	16	0.6
14	20	0.55	22	16	0.45
15	20	0.45	23	15	0.6

After running the SAS program these engines were ordered according to the predicted values of each model. They were arranged from lowest to highest for the thrust and highest to lowest for the fuel consumption and included engines 1 through 7 which were used to build the model. This arrangement allows the list to be split in half and only those engines appearing on the high thrust and low fuel consumption side were considered. The results, shown below, indicated that only engines number 15 and 16 were candidates with low fuel consumption and high thrust.

Thrust Model #2

Low High

9 7 8 23 12 11 10 21 13 6 14 | 5 <u>15</u> 19 <u>16</u> 2 4 17 1 22 18 3 20

Thrust Model #3

7 8 9 11 10 14 23 6 12 5 13 | <u>15</u> 21 <u>16</u> 2 4 1 19 22 18 17 3 20

Fuel Model #4

High Low

1 22 20 2 23 3 21 18 17 19 4 | <u>15</u> 12 5 9 <u>16</u> 14 11 6 7 10 13 8

Fuel Model #6

1 22 2 20 18 3 4 17 5 14 19 | 21 11 <u>15</u> 23 <u>16</u> 6 7 10 9 13 12 8

These engines were then "flown" using MISS to calculate the actual T_{SL} and fuel consumption. The results are shown below along with the best predicted value from the SAS results for models 2,3,4 and 6 in parentheses.

Engine	T_{SL} (lbf)	Fuel consumed
AAF	14,280	8199
15	14,509 (14,277)	8292 (8249)
16	14,308 (14,319)	8190 (8190)

This process quickly identified two acceptable engine designs. Engine 16 is better than the AAF engine for both thrust and fuel economy. Engine 15 gives a 229 lbf increase in thrust with a penalty in fuel economy of 93 lbs.

Comparing these engines against the constraint diagrams will determine if any real performance advantage has been achieved. This will be accomplished in Section 3.6 after the six-variable design analysis has been demonstrated.

3.5. Six-Variable Analysis

The two variable example showed the detailed procedure used to generate alternative design point engines. This next example was developed to show how the modeling technique can be used when considering more than two variables.

When only two variables were involved, surface plots could be generated of each model to display the effect of the two variables on the dependent variable. The problem is not as easily visualized for more than two variables and the information provided from the SAS program provided insight into the effect of each variable on the outcome.

A six variable analysis was completed to show the utility of the modeling technique when searching for the best set of design values. For this case the Mach number, altitude, π_c, α, π_c', and mass flow rate are the variables.

Proceeding as in the two variable analysis, choices of Mach number and altitude were made to begin the search. The decision was to concentrate on altitudes between 25,000 and 40,000 ft and Mach numbers between 1.4 and 1.7. This decision is quite arbitrary but it should reflect the leg of the mission where the aircraft will burn the most fuel.

Solution surfaces and trend analyses were performed at various combinations of Mach number and altitude. For example, Figure 9 in section 2.4.3 shows the solution surface for M=1.5 and 35,000 ft. This surface was based on the average value of $\pi_c'=3.7$ over the selected range of π_c and α. Figures 10 and 11 show the trend analyses and, from these three figures, the best range for π_c was selected to be between 19 and 24 with an α of 0.6. In this range the decrease in F/m_0 is reasonably constant while S continues to decrease.

From the design point ranges described above, a π_c of 19 and α of 0.6 were selected. The same mission described in section 3.3 and engine design performance values in Table 3 were used to calculate the fuel consumption and T_{SL} of the engine. After running MISS it was found that a mass flow of 105 lbm/s was adequate to fly the mission without requiring changes to the engine diameter. For this design point the result was T_{SL}=16,469 lbf and fuel consumption=8048 lbf.

After generating additional solution surfaces and trend analyses at various Mach numbers, altitudes, π_C, α, π_C', and mass flow, additional design points were selected. The choice of engine design points were selected at random as long as they fell on the solution surfaces and exhibited trend analyses which predicted good performance and low fuel consumption. The following design point engine information was collected:

Mach #	Alt (k ft)	π_C'	π_C	α	mass flow (lbm/s)	T_{SL} (lbf)	Fuel Req'd
1.4	25	3.6	22	0.6	130	14,161	8130
1.4	30	3.5	18	0.55	120	17,004	8118
1.4	35	3.7	17	0.7	110	18,288	7997
1.5	25	3.3	21	0.5	130	12,904	8268
1.5	30	3.6	20	0.5	115	14,972	8203
1.5	35	3.7	19	0.6	105	16,469	8048
1.5	40	3.7	20	0.6	90	17,653	7911
1.6	35	3.7	15	0.4	100	14,539	8579
1.6	35	3.7	19	0.6	100	14,308	8190
1.6	35	3.7	20	0.45	100	14,509	8292
1.6	35	3.7	22	0.65	100	12,855	8071
1.6	38	3.7	22	0.5	120	17,710	7895
1.7	30	3.4	18	0.5	120	12,820	8393
1.7	35	3.6	18	0.7	120	15,029	7784

Five models were constructed using the above data and the SAS results are presented in Appendix G. After reviewing the fit statistics, SAS models 2 and 3 for fuel consumption and models 4 and 5 for thrust were selected as the best fits. This selection was based on the F value, Prob>F, and the Root MSE. The Root MSE was used to judge the fit by looking for a low value which indicates small residuals. The following four SAS models resulted

$$Fuel_2 = 899.93M - 51.08h + 555.16\pi_c + 5563.41\alpha - 15.98\pi_c^2 - 6437.67\alpha^2 + 205.96(\pi_c')^2$$
$$Fuel_3 = 102.96M - 6.99h + 1136.83\pi_c - 36.02\pi_c^2 - 7577.5\alpha^2 + 17.38\pi_c^2\alpha^2 + 0.012\dot{m}^2$$
$$Thrust_4 = -9830.4 - 15210M + 609.07h + 2443.92\pi_c + 5084.65\alpha - 69.71\pi_c^2 - 7317.43\alpha^2 + 0.555\dot{m}^2$$
$$Thrust_5 = -12995 - 15495M + 623.38h + 2988.16\pi_c - 86.48\pi_c^2 - 6092.48\alpha^2 + 8.82\pi_c^2\alpha^2 + 0.582\dot{m}^2$$

where the subscript indicates the model number.

From these models it was not immediately obvious which variables significantly affect the dependent variables. This information was necessary to determine which direction to search for the candidate engine designs. More than one model was used to obtain a consensus for determining the influence and the SAS program provided the information needed.

The Prob>|T| statistic gave insight into the influence of each variable on the outcome. The closer this value was to 0.0001 the stronger the influence this variable has on the outcome. Arbitrarily selecting only those variables with Prob>|T| of 0.2 or below as showing significant influence in the model, the independent variables' effects can be determined. This determination was made by looking at both the model itself and the T for HO statistic.

The T for HO statistic indicates the dependence on the output by this variable: the larger it is in absolute value the greater the dependence. T for HO also indicates the direction of the influence on the dependent variable.

For the model, *Fuel*$_2$, there is dependence upon the Mach number, altitude, π_c^2, and $(\pi_c')^2$. The effect of each variable is summarized below where an up arrow indicates increase and a down arrow indicates a decrease.

$$M\uparrow \Rightarrow Fuel\uparrow$$
$$h\uparrow \Rightarrow Fuel\downarrow$$
$$\pi_c^2\uparrow \Rightarrow Fuel\downarrow$$
$$(\pi_c')^2\uparrow \Rightarrow Fuel\uparrow$$

These results show that for model 2, the fuel consumption can be decreased by lowering the Mach number, increasing the altitude, increasing the compressor pressure ratio, and decreasing the fan pressure ratio.

For the model, *Fuel3*, the following dependence was observed:

$$\pi_c \uparrow \Rightarrow Fuel \uparrow$$
$$\pi_c^2 \uparrow \Rightarrow Fuel \downarrow$$
$$\alpha^2 \uparrow \Rightarrow Fuel \downarrow$$
$$\pi_c^2 \alpha^2 \uparrow \Rightarrow Fuel \uparrow$$

There appears to be a contradiction of the effect of π_C on the output since π_C terms increase the fuel consumption and π_C^2 terms decrease the fuel consumption. In order to determine the effect of π_C the coefficients of the model are used to determine the influence of each of these terms subtracted from one another. In this case the tendency was toward an overall increase in the fuel consumption for an increase in π_C. The $\pi_C^2\alpha^2$ term has the effect that any increase in π_C or α will increase the fuel consumption.

The primary influences on models 4 and 5, *Thrust4* and *Thrust5*, are presented below. The overall effect of π_C for either model will be an increase in thrust.

Thrust4:

$$M \uparrow \Rightarrow T_{SL} \downarrow$$
$$h \uparrow \Rightarrow T_{SL} \uparrow$$
$$\pi_c \uparrow \Rightarrow T_{SL} \uparrow$$
$$\pi_c^2 \uparrow \Rightarrow T_{SL} \downarrow$$
$$\dot{m}^2 \uparrow \Rightarrow T_{SL} \uparrow$$

Thrust5:

$$M \uparrow \Rightarrow T_{SL} \downarrow$$
$$h \uparrow \Rightarrow T_{SL} \uparrow$$
$$\pi_c \uparrow \Rightarrow T_{SL} \uparrow$$
$$\pi_c^2 \uparrow \Rightarrow T_{SL} \downarrow$$
$$\dot{m}^2 \uparrow \Rightarrow T_{SL} \uparrow$$

As a result of this investigation, the effect of the design variables on the fuel consumption and T_{SL} can be summarized as follows

$$M \uparrow \Rightarrow Fuel \uparrow T_{SL} \downarrow$$
$$h \uparrow \Rightarrow Fuel \downarrow T_{SL} \uparrow$$
$$\pi_c \uparrow \Rightarrow Fuel \uparrow T_{SL} \uparrow$$
$$\alpha \uparrow \Rightarrow Fuel \downarrow$$
$$\dot{m} \uparrow \Rightarrow T_{SL} \uparrow$$
$$\pi_c' \uparrow \Rightarrow Fuel \uparrow$$

The Mach number, altitude, and compressor pressure ratio have the most effect on both the fuel consumption and T_{SL} whereas the bypass ratio, mass flow rate, and fan pressure ratio showed a stronger influence on either fuel consumption or T_{SL} but not both.

The best choice for seeking new design points with improved fuel economy and increased sea-level thrust would be at lower Mach numbers and fan pressure ratios, while raising the altitudes, bypass ratios and mass flows. Since π_c raises both T_{SL} and fuel consumption its influence would have to be checked at each design point Mach number and altitude to determine the best π_c as in the two variable analysis of section 3.4.

These models have produced sufficient trend data to select the next best design point away from the existing AAF baseline design point engine. The information gathered thus far considered only 14 design points and covered a wide range for the design variables. Using this information, along with the models and the results for the design point engines used to build the models, the new design points are selected.

The Mach numbers 1.4 and 1.5 were chosen since decreasing the Mach number will produce both a decrease in fuel consumption and an increase in T_{SL}. An increase in altitude will also improve overall performance but increasing the altitude too much can cause poor performance for lower altitude maneuvers, as in loiter phases. The altitude selected should not vary much from the original design point if it represents a good compromise between the high altitude requirement and the majority of the low altitude maneuvers. The value of 35,000 ft is still a good design point and will be retained.

Increasing π_c increases both the dependent variables and an average π_c (representing the range of π_c values used for the modeling) should provide good results. Selecting values of 18 and 19 for π_c will produce some variation in the engines for comparison with the candidate retrofit engine designs already discussed.

For α, a fairly high bypass ratio was selected to keep fuel consumption down. A value of 0.65 reflects an acceptable compromise among the design point engines used for the models. A mass flow rate of 110 lbm/s was chosen which represents a 10% increase over the existing retrofit designs.

Engine performance can be very sensitive to π_c' when negotiating a particular maneuver. It should be selected based on the results of the analysis at the design point Mach number and altitude. This was previously accomplished when collecting the data for the models and a value of 3.7 will be used.

These decisions resulted in the following proposed design points which were "flown" using MISS to determine their T_{SL} and fuel consumption:

Mach #	Altitude (k ft)	π_c'	π_c	α	mass flow* (lbm/s)	T_{SL} (lbf)	Fuel Req'd (lbf)
1.4	35	3.7	18	0.65	100	16,986	8056
1.4	35	3.7	19	0.65	100	17,056	8008
1.5	35	3.7	18	0.65	110	17,015	8010
1.5	35	3.7	19	0.65	110	17,050	7966

*It was found during the MISS analysis that the inlet had to be resized for M=1.4, therefore the mass flow was reduced to 100 lbm/s

These results showed that the modeling scheme determined the proper direction to move when selecting the next choice for each design variable. The thrust has been significantly increased over that of the AAF and fuel has been saved in the process. This was an excellent result considering the amount of data points used (18 including these last 4 engines) and the improvements achieved. Further refinement of these engines could be made by using only the M=1.4 and M=1.5 values, over small ranges of π_c, α, or any other design variable and constructing new models. This then becomes the same type of search as was presented in the two-variable example.

These engines will now be checked against the constraint/thrust diagrams to view the performance improvements from the two-variable search and the six-variable search. The new baseline engine will also be selected.

3.6. Engine Comparisons Using Constraint/Thrust Diagrams

The two-variable and six-variable analyses identified six candidate baseline engine designs for use in the retrofit effort. These engines were compared using the constraint/thrust diagrams generated in the section on constraint diagrams. The method used was to take the existing constraint/performance diagrams and determine the relative position of each engine on the plot. The M=0.9 turn, M=1.6 turn and supercruise flight conditions were compared.

The candidate baseline engines are summarized below for easy reference

Engine	Mach #	Altitude (k ft)	π_c	α	π_c'	mass flow (lbm/s)	T_{SL} (lbf)	Fuel Req'd (lbf)
A	1.6	35	20	0.55	3.7	100	14,280	8199
B	1.6	35	20	0.45	3.7	100	14,509	8292
C	1.6	35	19	0.6	3.7	100	14,308	8190
D	1.4	35	18	0.65	3.7	100	16,986	8056
E	1.4	35	19	0.65	3.7	100	17,056	8008
F	1.5	35	18	0.65	3.7	110	17,015	8010
G	1.5	35	19	0.65	3.7	110	17,050	7966

In order to judge the performance improvement of each engine, a line representing the AAF wing loading value of 64 was drawn on each of the constraint/thrust diagrams. The AAF is a two engine aircraft, therefore the sea-level thrust of each engine is multiplied by two for the total aircraft T_{SL}. By plotting the total sea-level thrust along this line, the performance advantage of one engine design over the other can be seen. The engines were broken up into two groups since the change in thrust among members of either group was not significant and represent the same point on the constraint/thrust diagram. These diagrams are presented in Figures 31, 32 and 33.

Figure 31 indicates that engines A, B, and C allow the AAF to negotiate a M=0.9 turn for a β of 0.77, whereas engines D, E, F, and G allow the maneuver at a β of 0.83. In comparison to the weight of the aircraft at this maneuver, the first set of engines produces enough thrust for the aircraft to maneuver at a weight of 18,788 lbf vs. 20,252 lbf for the second set. This is a significant improvement which would allow the aircraft to maneuver with additional fuel or munitions on board.

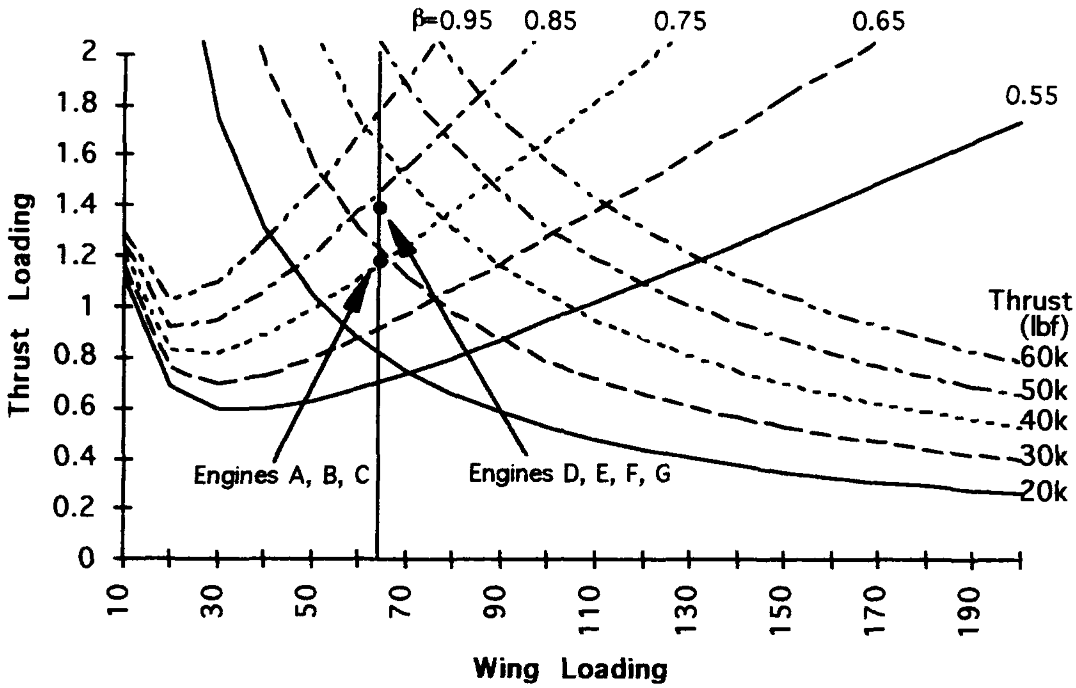


Figure 31. M=0.9 Turn Constraint/Thrust Diagram Showing Baseline Engine Thrust

Figure 32 presents a similar improvement. Engines D, E, F, and G will power the aircraft through a M=1.6, 5 g turn at a weight above 23,180 lbf (β=0.95), an improvement in weight of at least 1400 lbf over the other engines.

The supercruise requirement, Figure 33, indicated that the aircraft will easily perform supercruise at a β of 0.95 with engines D, E, F, and G. The other engines were barely adequate to perform this maneuver.

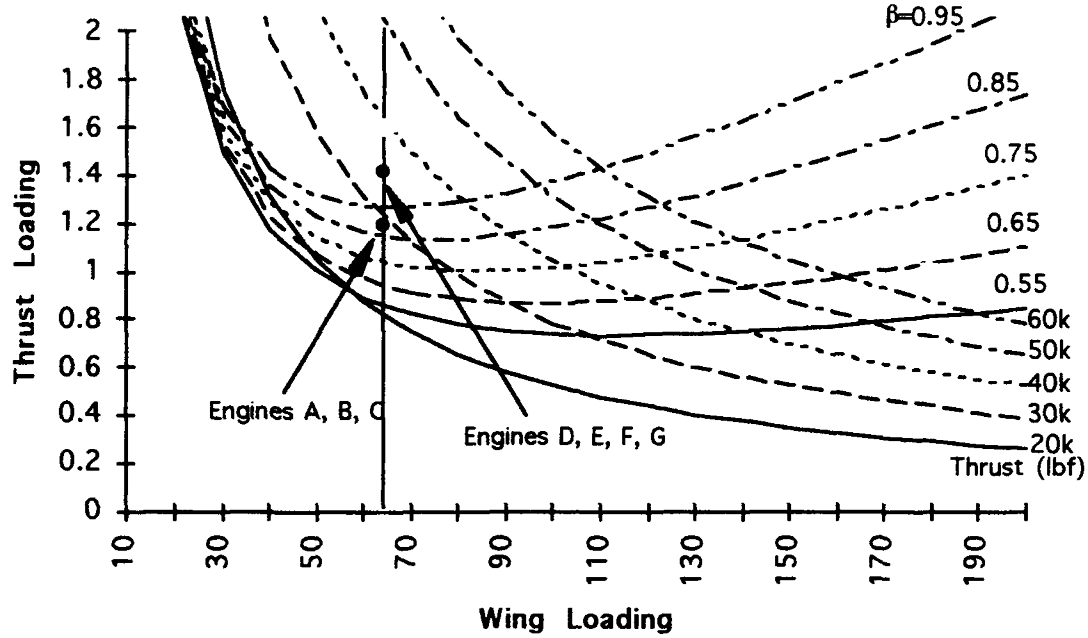


Figure 32. M=1.6 Turn Constraint/Thrust Diagram Showing Baseline Engine Thrust

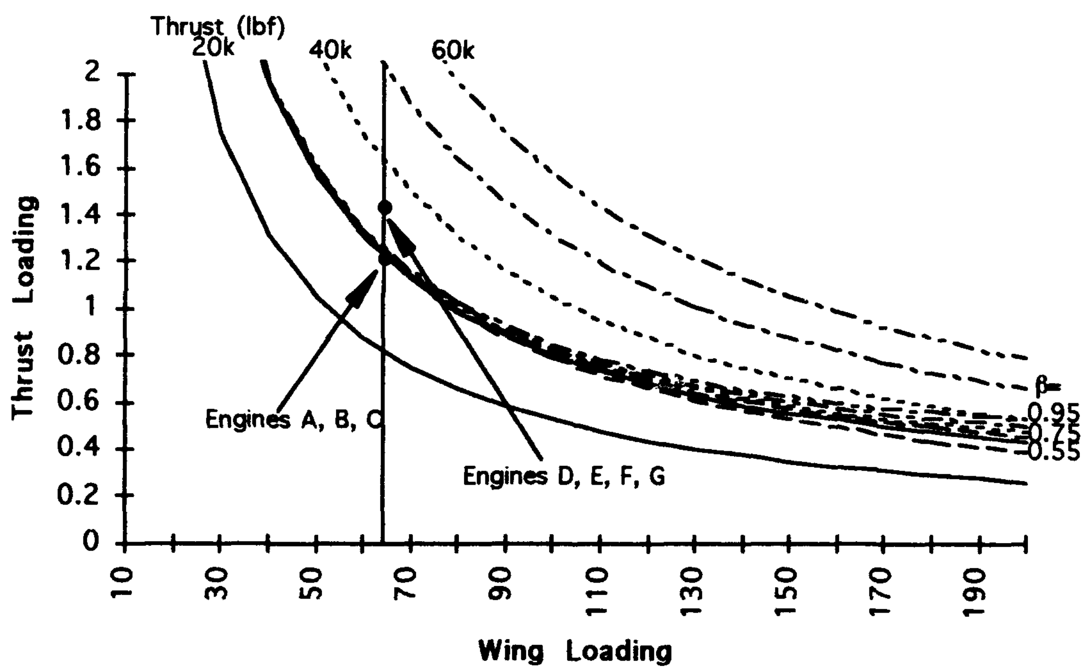


Figure 33. Supercruise Constraint/Thrust Diagram Showing Baseline Engine Thrust

Since the change in engine thrust within the groups was small , there is very little to compare. Each engine performed the mission and negotiated the maneuvers at the required weight fractions. The advantage is in the improvement which was made with the second group of engines.

The last issue to examine was the fuel consumption. Among the possible candidates, engines D, E, F, and G all showed improved fuel economy over engines A, B, and C. Therefore, these engines represent both an improvement in performance as well as an improvement in fuel economy.

From these results, the new baseline design point engine selected was engine G. This engine has the lowest fuel consumption of all candidates and a T_{SL} of only 6 lbf less than the highest T_{SL} in the group. The original AAF engine and the new engine are presented below for comparison.

Engine	Mach #	Altitude (k ft)	π_c	α	π_c'	mass flow (lbm/s)	T_{SL} (lbf)	Fuel Req'd (lbf)
AAF	1.6	35	20	0.55	3.7	100	14,280	8199
G	1.5	35	19	0.65	3.7	110	17,050	7966

Maximum maneuver weights were determined based on the intersections of the thrust line, the wing loading line, and the constraint line for the particular maneuver.

The application of empirical modeling and statistical analysis in the engine search greatly enhanced the ability to detect trends and make inferences between the engine design variables, the total fuel consumption, and sea-level thrust. This tool eliminates the need to test each new choice by producing a model by which to judge the influence of each independent variable on the dependent variable. The modeling technique quickly narrowed the choice of ranges for the design variables.

The modeling method required a minimum number of observations to produce acceptable results. The AAF design point engine analysis was based on only 7 observations and resulted in accurate results. The six-variable analysis used 14 observations and resulted in a rapid convergence on the influence of each design variable.

The two-variable analysis resulted in two new designs. The best of these improved the sea-level thrust by 0.2% and the fuel consumption by 0.1%. This indicated that the original AAF engine was nearly optimum for the design point conditions and very little improvement could be made.

The six-variable analysis covered a much wider range of design choices and resulted in a significant improvement over the AAF engine. The thrust was increased by 16% and the fuel consumption was reduced by 3%. The real advantage to the new engine design was the increase in thrust which enabled the aircraft to perform the same maneuvers at higher aircraft weight.

A final conclusion is that the design scheme presents a method which will reduce the amount of iteration by using the quality measures and modeling technique. A good model eliminates the need to test each new design point choice with MISS therefore reduces the amount of computer time required.

Further research into the modeling scheme should address the following areas:

1. The search for better models and the introduction of optimization techniques could greatly enhance the engine design process.

2. Incorporation of the effect of throttle ratio into this analysis scheme should be explored.

3. Integration of this procedure into the AFIT design course to provide feedback and test the design scheme on a wide range of aircraft engine retrofit scenarios. This could be accomplished by introducing the SAS modeling technique as applied specifically to engine design and following the design scheme outlined in this investigation to augment the textbook *Aircraft Engine Design*.

Appendix A: ACSYS Computer Program

The ACSYS program performs two major functions. It can perform a constraint analysis to generate constraint diagrams as well as produce P_S/f_S contour plots. This appendix contains excerpts from the ACSYS program user guide, Reference 5. It provides a brief overview of the program components used in this research.

The constraint analysis is based on the constraint equations of Chapter 2, Reference 6. This analysis was used extensively to generate the design data for constructing the constraint diagrams and constraint/thrust diagrams. The program is capable of analyzing nine constraint types, seven aircraft drag models, and six engine thrust models. Once the constraint analysis has been calculated the constraint lines can be plotted on the screen.

The program is organized with the following menus and sub-menus

- Constraint Analysis
  - Select Constraints
  - Constraint Data
  - Range of W/S and T/W
  - Perform Calculations
  - Plot Constraint Diagram
- P_S/f_S Contour Plot
  - Select Calculation Desired ($1=P_S$ or $2=f_S$)
  - Aircraft System Design Point
  - Range of Velocities and Altitudes
  - Calculate and Plot Contours
- Input Data (Read or Save Input Data File)
- Aircraft (Six Models plus One User Input Model)
- Engine Thrust (Four Models plus Two User Input Models)
- Engine TSFC (Four Models pluds One User Input Model)
- Output Devices (Screen, Printer, or Both)
- Screen Resolution (Hi Res B&W, CGA, EGA)
- Exit Program

The information needed to perform a constraint analysis is input to the program using the sub-menus which consist of the following options:

Constraints

A-Constant Altitude Speed Cruise
B-Constant Speed Climb
C-Constant Altitude Speed Turn
D-Horizontal Acceleration
E-Service Ceiling
F-Takeoff (No Obstacle)
G-Takeoff
H-Landing (No Obstacle)
I-Landing

Drag Models

–Cargo/Passenger-TProp (Low Drag)
–Cargo/Passenger-TProp (High Drag)
–Cargo/Passenger-TFan (Low Drag)
–Cargo/Passenger-TFan (High Drag)
–Future Fighter
–Current Fighter
–User Input Drag Model

Engine Thrust Models

–High Bypass Ratio Turbofan
–Low Bypass Ratio Mixed Turbofan
–Advanced Turbojet
–Advanced Turboprop
–User Input Thrust Model
–User Input Alpha Values

Engine TSFC Models

–High Bypass Ratio Turbofan
–Low Bypass Ratio Mixed Turbofan
–Advanced Turbojet
–Advanced Turboprop
–User Input TSFC Model, C=f(M)

Various information must be input for each constraint type to perform the calculation. The data item and constraint it corresponds to are summarized below.

Data Item	Constraint	Data Item	Constraint
- β (Weight Fraction)	A thru I	- dh/dt (ft/s)	B
- Temperature (R)	A thru I	- Number of g's	C
- Altitude (ft)	A thru I	- dV/dt (ft/s^2)	D
- CDR	A thru I	- P_s (ft/s)	E
- Velocity (ft/s)	A thru E	- Total landing distance (ft)	H, I
- Fraction of max thrust	A thru E	- Fraction C_{Lmax} for braking	H, I
- C_{Lmax}	F thru I	- k_{TD}	H, I
- Mach # during takeoff roll	F, G	- Thrust reverser (-α)	H, I
- Takeoff Friction Coefficient	F, G	- Free roll time (s)	H, I
- Total takeoff distance (ft)	F, G	- Braking friction coefficient	H, I
- Rotation time (s)	F, G	- Landing obstacle height (ft)	I
- k_{TO}	F, G	- kobs	I
- Takeoff obstacle height (ft)	G	- Engine thrust lapse (α)	A thru G (user input engine)
- Engine AB (1=ON)	A thru G		

After loading the appropraite information for each constraint, the program will generate the constraint data and the constraint diagram. To receive a printout of the constraint diagram, the program EGADMP.COM can be used. This is most convenient for producing a diagram for use in reports or comparison analysis.

Appendix B: MISS Computer Program

The MISS computer program performs a multitude of functions in analyzing the off-design performance of an engine. It can perform a mission analysis to obtain estimates of the aircraft's takeoff weight, fuel usage, and engine thrust. It may also be used to perform a thrust/drag analysis at a given Mach number and altitude to obtain variations in P_S and f_S. This appendix contains excerpts from the MISS program user guide, Reference 8. It provides a brief overview of the program components used in this research.

The program consists of the following menus and sub-menus:

MISSION:
- Define Legs
- View/Change Data
- Input/Save Data

SYSTEM:
- Aircraft Drag Model
- Engine Model
  - Engine Thrust Models
  - Engine TSFC Models
  - Engine Model 7
    - Number of Engines
    - Engine Operating Limits
    - Installation Loss Model
- Aircraft Drag Model

CALCULATION:
- Mission Analysis
- Thrust/Drag Analysis

OTHER:
- Select Output Devices
- Exit Program

The mission section asks the user to define the mission legs, of which there are thirteen types. The aircraft's mission profile is loaded into the program by ordering the legs of the mission as they are expected to be performed. The possible choices for mission legs are listed below

A-Constant Speed Climb
B-Horizontal Acceleration
C-Climb and Acceleration
D-Takeoff Acceleration
E-Constant Altitude/Speed Cruise
F-Constant Altitude/Speed Turn
G-Best Cruise Mach and Altitude
H-Loiter
I-Warm-up
J-Takeoff Rotation
K-Constant Energy Height Maneuver
L-Deliver Expendables
M-Descend (P_S=0)

Once the mission has been defined, the mission data for each leg is input.

Mission Data	Mission Type	Mission Data	Mission Type
- Altitude	B, D, E, F, H, I, J	- Fraction of avg vel in vertical	K
- Initial altitude	A, C, K, M	- Angle of descent	M
- Temperature	A, B, C, D, E, F, H, I, J, K, M	- k_{TO}	D, J
- Velocity	A, E, F, J	- Takeoff friction coefficient	D
- Initial velocity	B, C, K, M	- Rotation time	J
- Mach number	A, E, F, J	- Distance	E, G
- Initial Mach number	B, C, K, M	- Time	H, I
- Best cruise Mach number	G	- Legs included in distance calculation	E, G
- Final altitude	A, C, K, M	- Engine thrust lapse	A, B, C, D, F, I, J, K, M (user input α's)
- Final velocity	B, C, K, M	- Engine AB (1=ON)	A, B, C, D, F, I, J, K, M
- Final Mach number	B, C, K, M	- Engine TSFC	A thru K, M (user input)
- Number of g's	F	- Maximum T_{t4}	A thru K, M
- Avg Mach number	B, C, M	- Engine C	A thru K, M
- Number of turns	F	- Maximum T_{t7}	A thru K, M
- CDR	A thru K, M	- Payload expended	L
- C_{Lmax}	D, J	- Total inlet/nozzle installation loss	A thru K, M

The aircraft wing loading, thrust loading, weight fraction, and takeoff weight are input to define the particular aircraft design point. The aircraft and engine model is completed by selecting from the available options indicated below.

Drag Models

--Cargo/Passenger-TProp (Low Drag)
--Cargo/Passenger-TProp (High Drag)
--Cargo/Passenger-TFan (Low Drag)
--Cargo/Passenger-TFan (High Drag)
--Future Fighter
--Current Fighter
--User Input Drag Model

Engine Models

--Basic Engine Model (consists of a thrust model and TSFC model)

Engine Thrust Models	Engine TSFC Models
--High Bypass Ratio Turbofan	--High Bypass Ratio Turbofan
--Low Bypass Ratio Mixed Turbofan	--Low Bypass Ratio Mixed Turbofan
--Advanced Turbojet	--Advanced Turbojet
--Advanced Turboprop	--Advanced Turboprop
--User Input Thrust Model	--User Input Model
--User Input Alpha Values	--User Input TSFC or C Values

--Advanced Engine Model (reads a single point reference data file from ONX or OFFX)

The advanced engine model requires the user to set the number of engines, the engine operating limits, and the installation loss model. The engine operating limits, which are determined by the material properties and to ensure proper mixing at the burner, consist of

Max Compressor Pressure Ratio	Max % Ref RPM - LP Spool
Max Pressure at Station 3	Max % Ref RPM - HP Spool
Max Temperature at Station 3	

The installation loss model, which is made up of the following inputs, accounts for the inlet and nozzle dimensions and their associated losses. The inlet capture area, nozzle

area, and nozzle length may be input.

Constant loss for all mission legs
Different losses for each mission leg
Loss model of chapter 6

After all the mission, aircraft, and engine data have been loaded, the mission can be executed. The analysis provides a leg by leg accounting of the changes in the weight fraction of the aircraft as the fuel is burned. It provides an accounting for each leg of the engine's performance including the required thrust, losses, and temperatures, while warning the user if the off-design solution fails to converge or the inlet size is too small.

This program is most useful in evaluating an engine over a wide range of operating conditions. It also allows the student designer to try all types of engines and aircraft designs, compare how well they perform the mission, and make intelligent choices as to which is the best design combination.

Appendix C: SAS Software

In this investigation, the modeling tool used is multivariable linear regression with statistical analysis. This method takes a group of input and output data and attempts to build a model to fit the output based on the input conditions. The SAS software has been developed to provide the data analyst with a tool for applying various statistical analyses when building models or performing optimization studies. This appendix is based on the information in Reference 13.

The SAS program is a self contained analysis tool which uses a two step process for analyzing the data. The first step is a data step, wherein the data is organized. The second step is a process step, which tells SAS how to operate on the data (13:19). The program portion of the analysis implies a command file containing SAS instructions and data which is input to the SAS environment. This file is built by the analyst and reflects the type of analysis requested. A typical SAS program file for this paper is presented below.

```
TITLE 'MODELING MISSION FUEL CONSUMPTION AND THRUST AT SEA LEVEL';
DATA ENGINES;
      INPUT FUEL THRUST MACH ALT PIC ALPHA PICP MDOT
      PICSQ=PIC**2
      ALPHASQ=ALPHA**2
      B=PIC*ALPHA
      D=PIC**2*ALPHA**2
CARDS;
8579 14539 1.6 35 15 .4 3.7 100
8268 14375 1.6 35 18 .55 3.7 100
8143 13079 1.6 35 22 .55 3.7 100
8410 14400 1.6 35 16 .5 3.7 100
8159 14166 1.6 35 20 .6 3.7 100
8335 14452 1.6 35 17.5 .5 3.7 100
. . 1.6 35 22 .65 3.7 100
. . 1.6 35 20 .45 3.7 100
. . 1.6 35 16 .6 3.7 100
;
PROC PRINT;
PROC REG SIMPLE;
      MODEL THRUST=PIC ALPHA D/CLI NOINT;
      MODEL FUEL=PIC ALPHA PICSQ B/CLI;
```

The first line of the program produces a title for the output pages. The next section under DATA ENGINES; creates the order in which the dependent and independent variables will be read from the CARDS section and defines the terms used in the model. If a quadratic term of the variable PIC is required, then a new variable PICSQ=PIC**2 must be defined.

The CARDS section of the program contains the input data. This data is entered in the same order as defined in the portion of the program under DATA ENGINES. The PROC PRINT statement allows the output to be formatted for printing and the last section of the program contains the information for SAS to construct the model. The semicolon indicates the end of a SAS statement.

The process is PROC REG SIMPLE or simple regression analysis. Each model begins with the word MODEL. The value to be modeled is listed next with an equals sign indicating which variables the model should be based on. A slash indicates the end of the variables and the beginning of any special instructions to be used by SAS. The term CLI after the slash tells SAS to produce an output which includes the predicted values and the 95% prediction limits for the model. The last command, NOINT, tells SAS to build the model forcing the intercept through the origin. There will be no residual terms when all the input variables are entered as zero. If an intercept is desired delete the command.

The program output for a typical SAS analysis is presented at the end of this appendix (Note: SAS output pages 1 and 2 have been combined on one page, and SAS output pages 3 and 4 been combined on one page). Page 1 of SAS output shows the values for all the input variables. The second page provides the descriptive statistics for the variables including the sum, mean, uncorrected sum of squares, variance, and standard deviation.

Page 3 contains the output titled Analysis of Variance which provides statistical information for the model's fit to the data. The degrees of freedom for the model, DF, is equal to the number of variables used for the model. The DF of the error is the difference between the Model DF and the U Total DF, which is equal to the sample size for models without an intercept, or the C Total DF, which is one less than the sample size for models with an intercept.

The sum of squares, SS, separates the variation in the data into portions which can be attributed to the model and to the error. The sum of squares is equal to

$$SS = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

where y_i is the observed response and $\bar{y}$ is the mean response. The Total SS=Model SS+Error SS where the Error SS is the sum of the residuals squared.

The mean square, MS, is the sum of squares divided by the degrees of freedom. The Error MS estimates the variance of a population to a given set of independent variables.

The F Value is a test statistic which is calculated by taking the MS for the model and dividing it by the MS of the error. This statistic is the first indication of how good the model is. The value for F tests the hypothesis that the model explains a significant portion of the variation in the data.

$$F > F_\alpha$$

This is a test of the null hypothesis which, if the value of F is larger than the value of F_α, indicates that the mean of the data did not happen by chance. The value for F_α is found in tables and is based upon the DF for the model and the DF for the error and is written as $F(\alpha$, DF model, DF error) where α represents the error probability, normally 0.05.

The value Prob>F is the p-value associated with the same test and indicates whether or not the model explains a significant portion of the variation in the data. If the choice for α is greater than the value of p then the null hypothesis is rejected. This measure provides an indication of the extent to which the observed data disagrees with the null hypothesis. If the p-value is 0.0001 then the model is called a significant model.

The Root MSE is equal to the square root of the Error SS divided by the DF of the error or, the square root of the mean squared error, MSE. The Dep Mean, is simply the mean value of the dependent variable. The value C.V. is the coefficient of variation and is calculated by taking the Root MSE divided by the dependent mean and multiplying

by 100. This is the Root MSE expressed as a percentage of the dependent mean. The smaller this value is the smaller the Root MSE and the smaller the residuals of the model.

R-Square is the coefficient of multiple determination and represents the fraction of the total variation due to the variables in the model and can range from 0 to 1. It represents the correlation between y and $\hat{y}$ and is equal to

$$RSquare = \frac{\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

where $\hat{y}_i$ is the approximate response. It is equal to the Model SS/Total SS and the closer this value is to 1, the better the model is at accounting for variation in the data.

The next section of the SAS output presents the parameter estimates and gives the coefficients for each of the independent variables and indicates their relative importance in the model.

The degree of freedom for each parameter should be 1. The standard error is an indicator for how much the parameter estimates would vary from one set of data to the next. This can be used to attain confidence intervals around the parameter estimates.

The T for HO: Parameter=0, presents the t-value for testing the null hypothesis that the parameter is 0 using the t-distribution. The t-value is calculated by taking the parameter estimate and dividing by the standard error. If there is very little dependence on the output by this variable, then this value will be small indicating it can be dropped from the analysis. Another important analysis tool to determine the relative worth of a particular variable is by using the Prob>|T| which gives the p-value for the t-value of the parameter. If this value is close to 1 then it indicates that the slope of the data is zero for that parameter. In other words there is very little change to the data based upon that parameter. If the value is close to 0.0001 it indicates that the slope is not zero and that the data is dependent upon that variable.

The last page of SAS output summarizes the model by presenting the dependent variable, the predicted value, standard error predicted, lower and upper 95% predicted value, and the residuals. Most of these columns are self explanatory. The Standard Err Predict is a measure of how much the predicted value estimate would vary from one set of data to the next. The Lower and Upper 95% Predict gives the prediction limits. This means that you can be 95% confident that the actual value lies somewhere between the upper and lower values. The Residual is the error between the predicted and the actual value.

Once a model has been built it is easy to obtain new predictions for different input variables. This is accomplished by placing a period in place of the dependent variable and listing the new input variables the same way as the model data. The predicted values and their confidence interval can be used to make inferences about new values for design variables. For this model, observations 7, 8, and 9 show predictions based on the input conditions.

There are additional tests which may be used to determine how well the regression model fits the data. These tests are explained in detail in Chapter 6 of Reference 2 and Chapter 10 of Reference 13.

OBS	FUEL	THRUST	MACH	ALT	PIC	ALPHA	PICP	MDOT	PICSQ
1	8579	14539	1.6	35	15.0	0.40	3.7	100	225.00
2	8268	14375	1.6	35	18.0	0.55	3.7	100	324.00
3	8143	13079	1.6	35	22.0	0.55	3.7	100	484.00
4	8410	14400	1.6	35	16.0	0.50	3.7	100	256.00
5	8159	14166	1.6	35	20.0	0.60	3.7	100	400.00
6	8335	14452	1.6	35	17.5	0.50	3.7	100	306.25
7	.	.	1.6	35	22.0	0.65	3.7	100	484.00
8	.	.	1.6	35	20.0	0.45	3.7	100	400.00
9	.	.	1.6	35	16.0	0.60	3.7	100	256.00

OBS	ALPHASQ	B	D
1	0.1600	6.00	36.00
2	0.3025	9.90	98.01
3	0.3025	12.10	146.41
4	0.2500	8.00	64.00
5	0.3600	12.00	144.00
6	0.2500	8.75	76.563
7	0.4225	14.30	204.49
8	0.2025	9.00	81.00
9	0.3600	9.60	92.16

****

Descriptive Statistics

Variables	Sum	Mean	Uncorrected SS
INTERCEP	6	1	6
PIC	108.5	18.083333333	1995.25
ALPHA	3.1	0.5166666667	1.625
D	564.9825	94.16375	63031.664606
THRUST	85011	14168.5	1205979247
PICSQ	1995.25	332.54166667	709182.0625
B	56.75	9.4583333333	564.9825
FUEL	49894	8315.6666667	415037120

Variables	Variance	Std Deviation
INTERCEP	0	0
PIC	6.641666667	2.5771431211
ALPHA	0.004666667	0.0683130051
D	1966.1587444	44.341388616
THRUST	300178.7	547.88566324
PICSQ	9135.6604167	95.580648756
B	5.6444166667	2.3757981115
FUEL	27049.466667	164.46722064

Model: MODEL1
NOTE: No intercept in model. R-square is redefined
Dependent Variable: THRUST

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	3	1205007724.5	401669241.51	1240.329	0.0001
Error	3	971522.45528	323840.81843		
U Total	6	1205979247			

Root MSE	569.07013	R-Square	0.9992
Dep Mean	14168.50000	Adj R-sq	0.9984
C.V.	4.01645		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob>ITI
PIC	1	515.240497	167.1484383	3.083	0.0540
ALPHA	1	21711	5486.284904	3.957	0.0288
D	1	-67.685055	8.17087135	-8.284	0.0037

****

Obs	Dep Var THRUST	Predict Value	Std Err Predict	Lower 95% Predict	Upper 95% Predict	Residual
1	14539.0	13976.2	452.098	11663.1	16289.2	562.8
2	14375.0	14581.3	310.633	12518.0	16644.6	-206.3
3	13079.0	13366.3	531.121	10889.0	15843.6	-287.3
4	14400.0	14767.3	354.911	12632.9	16901.7	-367.3
5	14166.0	13584.5	437.889	11299.3	15869.7	581.5
6	14452.0	14689.8	266.157	12690.5	16689.2	-237.8
7	.	11606.2	779.265	8535.3	14677.1	.
8	.	14592.1	754.641	11584.1	17600.0	.
9	.	15032.3	827.072	11837.3	18227.3	.

Sum of Residuals	45.62706
Sum of Squared Residuals	971522.4553
Predicted Resid SS (Press)	9867627.6973

Appendix D: MISS Analysis Data

This appendix contains an example result from a mission analysis. This example is representative of all the MISS program calculations used in this paper. From this data the particular attributes assigned for each mission leg can be determined.

```
              MISSION ANALYSIS CALCULATIONS
     MISSION DATA FILE: MISSION5.DAT
     MODELS: AIRCRAFT # 5    THRUST # 7    TSFC # 7
     ENGINE DATA FILE: AAFSP100.DAT
         NUMBER OF ENGINES                      2
         MAX PRESSURE AT STATION 3          350.0 psia
         MAX TEMPERATURE AT STATION 3      1760.0 R
         MAX % REF RPM - LP SPOOL           110.0 %
         MAX % REF RPM - HP SPOOL           110.0 %
         Area 1 =  3.29  Area 10 =  4.03  N Lngth =  4.09
     CURRENT SELECTION:
         WING LOADING (W/S)                 64.00 psf
         THRUST LOADING (T/W)                1.20
         WEIGHT FRACTION (BETA)            1.0000
         AIRCRAFT GROSS TAKEOFF WEIGHT      24400.lb

MISSION LEG # 1  NAME: WARMUP TYPE: I - WARM-UP
           1 - Altitude (ft)  - - - - - - - - -    2000.
           2 - Temperature (R)  - - - - - - - -  511.60
               Speed of Sound (A0 - ft/sec) -  1108.7
          10 - CDR   - - - - - - - - - - - - -    .0000
          16 - Time (sec)   - - - - - - - - - -    60.0
          18 - Engine AB (1=ON/0=OFF) - - - -       0
          19 - Maximum TT4 (R)  - - - - - - - -   3200.
          ---------------- RESULTS ----------------
          Thrust    =  17059.lb       TSFC      =  .9892 1/hr
          TT4       =    3017.R       Limit     =  %RPM LP Spool
          Area 0*   =    2.60 ft2     Area 9    =   1.56 ft2
          IMS Noz   =  .1420          CD Noz    =  .0005
          Phi I     =  .1025          Phi N     =  .0000
          C         =  .9960 1/hr     PI        =  .9885
          Beta  1   = 1.0000          Beta  2   =  .9885

MISSION LEG # 2  NAME: TOACC  TYPE: D - TAKEOFF ACCELERATION
           1 - Altitude (ft)  - - - - - - - - -    2000.
           2 - Temperature (R)  - - - - - - - -  511.60
               Speed of Sound (A0 - ft/sec) -  1108.7
           4 - Mach Number  - - - - - - - - - -   .1800
          10 - CDR   - - - - - - - - - - - - -    .0700
          11 - CL max   - - - - - - - - - - - -  2.0000
          12 - kTO   - - - - - - - - - - - - -   1.2000
          13 - TO Friction Coeff  - - - - - -     .0500
          18 - Engine AB (1=ON/0=OFF) - - - -       1
          19 - Maximum TT4 (R)  - - - - - - - -   3200.
          20 - Maximum TT7 (R)  - - - - - - - -   3600.
          ---------------- RESULTS ----------------
          TO Mach   =  .1819          TO Vel    =  201.6 ft/sec
          Distance  =    669. ft      Distance  =  .1101 n mi
          CL        = 1.3889          CD        = .46780
          Etta TO   =  .3984          Thrust    =  27601.lb
          TT4       =    3031.R       Limit     =  %RPM LP Spool
          TT7       =    3600.R
          Area 0*   =    2.60 ft2     Area 9    =   2.48 ft2
          IMS Noz   =  .0462          CD Noz    =  .0002
          Phi I     =  .0296          Phi N     =  .0000
          u         =  .2943          C         = 1.8394 1/hr
          TSFC      = 1.8267 1/hr     PI        =  .9955
          Beta  2   =  .9885          Beta  3   =  .9840
```

```
MISSION LEG # 3  NAME: TOROT  TYPE: J - TAKEOFF ROTATION
           1 - Altitude (ft)  - - - - - - - - -    2000.
           2 - Temperature (R)  - - - - - - - -  511.60
               Speed of Sound (A0 - ft/sec) -  1108.7
          14 - Rotation Time - tR (sec)    - -      3.0
          18 - Engine AB (1=ON/0=OFF) - - - -        1
          19 - Maximum TT4 (R)  - - - - - - - -    3200.
          20 - Maximum TT7 (R)  - - - - - - - -    3600.
          ---------------- RESULTS ----------------
          Distance =    605. ft       Distance =  .0995 n mi
          Thrust   =  27608.lb        TSFC     = 1.8269 1/hr
          TT4      =    3031.R        Limit    =  %RPM LP Spool
          TT7      =    3600.R
          Area 0*  =    2.60 ft2      Area 9   =   2.48 ft2
          IMS Noz  =  .0462           CD Noz   =  .0002
          Phi I    =  .0292           Phi N    =  .0000
          C        = 1.8395 1/hr      PI       =  .9982
          Beta  3  =  .9840           Beta  4  =  .9823
```

```
MISSION LEG # 4  NAME: ACC     TYPE: B - HORIZONTAL ACCELERATION
           1 - Altitude (ft)  - - - - - - - - -    2000.
           2 - Temperature (R)  - - - - - - - -  511.60
               Speed of Sound (A0 - ft/sec) -  1108.7
           3 - Initial Velocity (ft/sec)  - -   200.9
           4 - Initial Mach Number  - - - - -   .1812
           6 - Final Velocity (ft/sec)  - - -   811.7
           7 - Final Mach Number  - - - - - -   .7000
          10 - CDR    - - - - - - - - - - - - -   .0000
          18 - Engine AB (1=ON/0=OFF) - - - -        0
          19 - Maximum TT4 (R)  - - - - - - - -    3200.
          ---------------- RESULTS ----------------
          Avg Mach =  .4406           Avg Vel  =  488.5 ft/sec
          Time     =    32.6 sec      Distance =   2.62 n mi
          CL       =  .2351           CD       = .02306
          CD/CL    =  .0981           Thrust   =  16835.lb
          TT4      =    3031.R        Limit    =  Pt3 Max
          Area 0*  =    2.47 ft2      Area 9   =   1.60 ft2
          IMS Noz  =  .1357           CD Noz   =  .0005
          Phi I    =  .0015           Phi N    =  .0000
          u        =  .1396           C        = 1.0614 1/hr
          TSFC     = 1.0541 1/hr      PI       =  .9933
          Beta  4  =  .9823           Beta  5  =  .9758
```

```
MISSION LEG # 5  NAME: C&A1   TYPE: C - CLIMB AND ACCELERATION
           1 - Initial Altitude (ft)  - - - -   2000.
           2 - Temperature (R)  - - - - - - -  511.60
               Speed of Sound (A0 - ft/sec) -  1108.7
           3 - Initial Velocity (ft/sec)  - -   776.1
           4 - Initial Mach Number  - - - - -   .7000
           5 - Final Altitude (ft)  - - - - -   9000.
           6 - Final Velocity (ft/sec)  - - -   897.5
           7 - Final Mach Number  - - - - - -   .8300
           8 - Average Mach Number  - - - - -   .7700
          10 - CDR  - - - - - - - - - - - - -   .0000
          18 - Engine AB (1=ON/0=OFF) - - - -       0
          19 - Maximum TT4 (R)  - - - - - - -   3200.
          ---------------- RESULTS ----------------
          Avg Mach =  .7700          Avg Vel  =  843.2 ft/sec
          Time     =   26.4 sec      Distance =   3.66 n mi
          Delta ze =  10157 ft
          CL       =  .0871          CD       = .01352
          CD/CL    =  .1553          Thrust   =  14567.lb
          TT4      =   3015.R        Limit    =  Pt3 Max
          Area 0*  =   2.27 ft2      Area 9   =   1.72 ft2
          IMS Noz  =  .1202          CD Noz   =  .0004
          Phi I    =  .0378          Phi N    =  .0001
          u        =  .2538          C        = 1.2272 1/hr
          TSFC     = 1.2038 1/hr     PI       =  .9946
          Beta  5  =  .9758          Beta  6  =  .9705

MISSION LEG # 6  NAME: C&A2   TYPE: C - CLIMB AND ACCELERATION
           1 - Initial Altitude (ft)  - - - -   9000.
           2 - Temperature (R)  - - - - - - -  486.60
               Speed of Sound (A0 - ft/sec) -  1081.3
           3 - Initial Velocity (ft/sec)  - -   897.5
           4 - Initial Mach Number  - - - - -   .8300
           5 - Final Altitude (ft)  - - - - -  16000.
           6 - Final Velocity (ft/sec)  - - -   895.3
           7 - Final Mach Number  - - - - - -   .8500
           8 - Average Mach Number  - - - - -   .8400
          10 - CDR  - - - - - - - - - - - - -   .0000
          18 - Engine AB (1=ON/0=OFF) - - - -       0
          19 - Maximum TT4 (R)  - - - - - - -   3200.
          ---------------- RESULTS ----------------
          Avg Mach =  .8400          Avg Vel  =  896.6 ft/sec
          Time     =   16.3 sec      Distance =   2.41 n mi
          Delta ze =   6939 ft
          CL       =  .0953          CD       = .01488
          CD/CL    =  .1562          Thrust   =  14915.lb
          TT4      =   3143.R        Limit    =  %RPM LP Spool
          Area 0*  =   2.54 ft2      Area 9   =   1.92 ft2
          IMS Noz  =  .0953          CD Noz   =  .0180
          Phi I    =  .0235          Phi N    =  .0062
          u        =  .2480          C        = 1.2397 1/hr
          TSFC     = 1.1853 1/hr     PI       =  .9966
          Beta  6  =  .9705          Beta  7  =  .9672
```

```
MISSION LEG # 7  NAME: C&A3   TYPE: C - CLIMB AND ACCELERATION
           1 - Initial Altitude (ft)  - - - -   16000.
           2 - Temperature (R)  - - - - - - -  461.70
               Speed of Sound (A0 - ft/sec) -  1053.3
           3 - Initial Velocity (ft/sec)  - -   895.3
           4 - Initial Mach Number  - - - - -   .8500
           5 - Final Altitude (ft)  - - - - -   23000.
           6 - Final Velocity (ft/sec)  - - -   851.9
           7 - Final Mach Number  - - - - - -   .8800
           8 - Average Mach Number  - - - - -   .8650
          10 - CDR   - - - - - - - - - - - -   .0000
          18 - Engine AB (1=ON/0=OFF) - - - -       0
          19 - Maximum TT4 (R)  - - - - - - -    3200.
          ----------------- RESULTS ----------------
          Avg Mach =  .8650          Avg Vel  =  898.7 ft/sec
          Time     =   17.6 sec      Distance =   2.60 n mi
          Delta ze =   5821 ft
          CL       =  .1189          CD       = .01656
          CD/CL    =  .1393          Thrust   =  11964.lb
          TT4      =   3043.R        Limit    =  %RPM LP Spool
          Area 0*  =   2.59 ft2      Area 9   =   1.98 ft2
          IMS Noz  =  .0894          CD Noz   =  .0177
          Phi I    =  .0225          Phi N    =  .0060
          u        =  .2747          C        = 1.2485 1/hr
          TSFC     = 1.1619 1/hr     PI       =  .9971
          Beta   7 =  .9672          Beta   8 =  .9644

MISSION LEG # 8  NAME: C&A4   TYPE: C - CLIMB AND ACCELERATION
           1 - Initial Altitude (ft)  - - - -   23000.
           2 - Temperature (R)  - - - - - - -  436.80
               Speed of Sound (A0 - ft/sec) -  1024.5
           3 - Initial Velocity (ft/sec)  - -   901.5
           4 - Initial Mach Number  - - - - -   .8800
           5 - Final Altitude (ft)  - - - - -   30000.
           6 - Final Velocity (ft/sec)  - - -   895.4
           7 - Final Mach Number  - - - - - -   .9000
           8 - Average Mach Number  - - - - -   .8900
          10 - CDR   - - - - - - - - - - - -   .0000
          18 - Engine AB (1=ON/0=OFF) - - - -       0
          19 - Maximum TT4 (R)  - - - - - - -    3200.
          ----------------- RESULTS ----------------
          Avg Mach =  .8900          Avg Vel  =  898.7 ft/sec
          Time     =   27.4 sec      Distance =   4.05 n mi
          Delta ze =   6828 ft
          CL       =  .1511          CD       = .01897
          CD/CL    =  .1255          Thrust   =   9481.lb
          TT4      =   2941.R        Limit    =  %RPM LP Spool
          Area 0*  =   2.65 ft2      Area 9   =   2.04 ft2
          IMS Noz  =  .0833          CD Noz   =  .0172
          Phi I    =  .0210          Phi N    =  .0058
          u        =  .3115          C        = 1.2576 1/hr
          TSFC     = 1.1375 1/hr     PI       =  .9965
          Beta   8 =  .9644          Beta   9 =  .9611
```

```
MISSION LEG # 9  NAME: SUBCRS TYPE: E - CONSTANT ALTITUDE/SPEED CRUISE
          1 - Altitude (ft)  - - - - - - - - -   30000.
          2 - Temperature (R)  - - - - - - - -  411.90
              Speed of Sound (A0 - ft/sec) -   994.9
          3 - Velocity (ft/sec)  - - - - - - -   895.4
          4 - Mach Number  - - - - - - - - - -   .0000
         10 - CDR  - - - - - - - - - - - - - -   .0000
         15 - Distance (Nautical Miles)  - -   150.0
         16 - Distance includes all prior
              mission legs beginning with leg      9
         18 - Engine AB (1-ON/0-OFF) - - - -       0
         19 - Maximum TT4 (R)  - - - - - - - -    3200.
         ---------------- RESULTS ----------------
         Distance -  150.0 n mi
         CL       -  .1723         CD        - .02060
         Thrust   -   2804.lb      TSFC      - 1.4824 1/hr
         TT4      -   2214.R       Limit     -  Thrust
         Area 0*  -   1.77 ft2     Area 9    -   1.56 ft2
         IMS Noz  -  .1427         CD Noz    -  .0170
         Phi I    -  .2186         Phi N     -  .0134
         C        - 1.6635 1/hr    PI        -  .9511
         Beta  9  -  .9611         Beta 10   -  .9141

MISSION LEG #10  NAME: PATROL TYPE: H - LOITER
          1 - Altitude (ft)  - - - - - - - - -   30000.
          2 - Temperature (R)  - - - - - - - -  411.90
              Speed of Sound (A0 - ft/sec) -   994.9
         10 - CDR  - - - - - - - - - - - - - -   .0000
         16 - Time (sec)  - - - - - - - - - -  1200.0
         18 - Engine AB (1-ON/0-OFF) - - - -       0
         19 - Maximum TT4 (R)  - - - - - - - -    3200.
         ---------------- RESULTS ----------------
         Mach     -  .7361         Vel       -  732.3 ft/sec
         CL       -  .2449         CD        - .02400
         Thrust   -   2185.lb      TSFC      - 1.3625 1/hr
         TT4      -   2016.R       Limit     -  Thrust
         Area 0*  -   1.65 ft2     Area 9    -   1.40 ft2
         IMS Noz  -  .1677         CD Noz    -  .0006
         Phi I    -  .1764         Phi N     -  .0003
         C        - 1.5289 1/hr    PI        -  .9565
         Beta 10  -  .9141         Beta 11   -  .8743
```

```
MISSION LEG #11  NAME: PENACC TYPE: B - HORIZONTAL ACCELERATION
          1 - Altitude (ft)  - - - - - - - -   30000.
          2 - Temperature (R)  - - - - - - -  411.90
              Speed of Sound (A0 - ft/sec) -   994.9
          3 - Initial Velocity (ft/sec)  - -   895.4
          4 - Initial Mach Number  - - - - -   .9000
          6 - Final Velocity (ft/sec)  - - -  1492.3
          7 - Final Mach Number  - - - - - -  1.5000
         10 - CDR  - - - - - - - - - - - - -   .0000
         18 - Engine AB (1=ON/0=OFF) - - - -       1
         19 - Maximum TT4 (R)  - - - - - - -    3200.
         20 - Maximum TT7 (R)  - - - - - - -    3600.
         ---------------- RESULTS ----------------
         Avg Mach = 1.2000          Avg Vel  = 1193.8 ft/sec
         Time     =   29.4 sec      Distance =   5.76 n mi
         CL       =  .0882          CD       = .02422
         CD/CL    =  .2748          Thrust   =  19344.lb
         TT4      =   3106.R        Limit    =  %RPM LP Spool
         TT7      =   3600.R
         Area 0   =   2.62 ft2      Area 9   =   3.82 ft2
         IMS Noz  =  .0007          CD Noz   =  .1160
         Phi I    =  .0115          Phi N    =  .0294
         u        =  .3030          C        = 2.1302 1/hr
         TSFC     = 1.8983 1/hr     PI       =  .9861
         Beta 11  =  .8743          Beta 12  =  .8621

MISSION LEG #12  NAME: PEN     TYPE: E - CONSTANT ALTITUDE/SPEED CRUISE
          1 - Altitude (ft)  - - - - - - - -   30000.
          2 - Temperature (R)  - - - - - - -  411.90
              Speed of Sound (A0 - ft/sec) -   994.9
          3 - Velocity (ft/sec)  - - - - - -  1492.3
          4 - Mach Number  - - - - - - - - -  1.5000
         10 - CDR  - - - - - - - - - - - - -   .0000
         15 - Distance (Nautical Miles)  - -   100.0
         16 - Distance includes all prior
              mission legs beginning with leg    12
         18 - Engine AB (1=ON/0=OFF) - - - -       0
         19 - Maximum TT4 (R)  - - - - - - -    3200.
         ---------------- RESULTS ----------------
         Distance =  100.0 n mi
         CL       =  .0556          CD       = .02868
         Thrust   =  10846.lb       TSFC     = 1.3505 1/hr
         TT4      =   3138.R        Limit    =  Thrust
         Area 0   =   2.48 ft2      Area 9   =   2.69 ft2
         IMS Noz  =  .0332          CD Noz   =  .0055
         Phi I    =  .0726          Phi N    =  .0012
         C        = 1.5154 1/hr     PI       =  .9242
         Beta 12  =  .8621          Beta 13  =  .7968
```

```
MISSION LEG #13  NAME: TURN1  TYPE: F - CONSTANT ALTITUDE/SPEED TURN
          1 - Altitude (ft)  - - - - - - - -    30000.
          2 - Temperature (R)  - - - - - - -  411.90
              Speed of Sound (A0 - ft/sec) -    994.9
          3 - Velocity (ft/sec)  - - - - - -  1591.8
          4 - Mach Number  - - - - - - - - -  1.6000
          8 - Number of G's  - - - - - - - -       5.0
          9 - Number of Turns  - - - - - - -       1.0
         10 - CDR     - - - - - - - - - - - -    .0000
         18 - Engine AB (1=ON/0=OFF) - - - -         1
         19 - Maximum TT4 (R)  - - - - - - -     3200.
         20 - Maximum TT7 (R)  - - - - - - -     3600.
         ---------------- RESULTS ----------------
         CL        =  .2260         CD       = .04298
         Thrust    =  18491.1b      TSFC     = 1.7183 1/hr
         TT4       =   3200.R       Limit    =  Thrust
         TT7       =   2750.R
         Area 0    =   2.54 ft2     Area 9   =   3.75 ft2
         IMS Noz   =  .0012         CD Noz   =  .0002
         Phi I     =  .0517         Phi N    =  .0000
         C         = 1.9283 1/hr    PI       =  .9716
         Beta 13   =  .7968         Beta 14  =  .7741

MISSION LEG #14  NAME: TURN2  TYPE: F - CONSTANT ALTITUDE/SPEED TURN
          1 - Altitude (ft)  - - - - - - - -    30000.
          2 - Temperature (R)  - - - - - - -  411.90
              Speed of Sound (A0 - ft/sec) -    994.9
          3 - Velocity (ft/sec)  - - - - - -    895.4
          4 - Mach Number  - - - - - - - - -    .9000
          8 - Number of G's  - - - - - - - -       5.0
          9 - Number of Turns  - - - - - - -       2.0
         10 - CDR     - - - - - - - - - - - -    .0000
         18 - Engine AB (1=ON/0=OFF) - - - -         1
         19 - Maximum TT4 (R)  - - - - - - -     3200.
         20 - Maximum TT7 (R)  - - - - - - -     3600.
         ---------------- RESULTS ----------------
         CL        =  .6939         CD       = .11096
         Thrust    =  15103.1b      TSFC     = 1.8187 1/hr
         TT4       =   2889.R       Limit    =  %RPM LP Spool
         TT7       =   3464.R
         Area 0*   =   2.68 ft2     Area 9   =   3.27 ft2
         IMS Noz   =  .0096         CD Noz   =  .0170
         Phi I     =  .0111         Phi N    =  .0032
         C         = 2.0409 1/hr    PI       =  .9716
         Beta 14   =  .7741         Beta 15  =  .7521
```

```
MISSION LEG #15  NAME: ACC    TYPE: B - HORIZONTAL ACCELERATION
          1 - Altitude (ft)  - - - - - - - - -   30000.
          2 - Temperature (R)  - - - - - - - -  411.90
              Speed of Sound (A0 - ft/sec) -   994.9
          3 - Initial Velocity (ft/sec)  - -   795.9
          4 - Initial Mach Number  - - - - -   .8000
          6 - Final Velocity (ft/sec)  - - -  1591.8
          7 - Final Mach Number  - - - - - -  1.6000
         10 - CDR   - - - - - - - - - - - -    .0000
         18 - Engine AB (1=ON/0=OFF) - - - -       1
         19 - Maximum TT4 (R)  - - - - - - -    3200.
         20 - Maximum TT7 (R)  - - - - - - -    3600.
         ---------------- RESULTS ----------------
         Avg Mach = 1.2000          Avg Vel  = 1193.8 ft/sec
         Time     =   33.4 sec      Distance =   6.56 n mi
         CL       =  .0758          CD       = .02382
         CD/CL    =  .3141          Thrust   =  19344.lb
         TT4      =   3106.R        Limit    =  %RPM LP Spool
         TT7      =   3600.R
         Area 0   =   2.62 ft2      Area 9   =   3.82 ft2
         IMS Noz  =  .0007          CD Noz   =  .1160
         Phi I    =  .0115          Phi N    =  .0294
         u        =  .2979          C        = 2.1302 1/hr
         TSFC     = 1.8983 1/hr     PI       =  .9816
         Beta 15  =  .7521          Beta 16  =  .7383

MISSION LEG #16  NAME: DELEXP TYPE: L - DELIVER EXPENDABLES
         21 - Payload Expended (lb)  - - - -    1309.
         ---------------- RESULTS ----------------
         Beta 16  =  .7383          Beta 17  =  .6846

MISSION LEG #17  NAME: ESC    TYPE: E - CONSTANT ALTITUDE/SPEED CRUISE
          1 - Altitude (ft)  - - - - - - - - -   30000.
          2 - Temperature (R)  - - - - - - - -  411.90
              Speed of Sound (A0 - ft/sec) -   994.9
          3 - Velocity (ft/sec)  - - - - - -  1492.3
          4 - Mach Number  - - - - - - - - -  1.5000
         10 - CDR   - - - - - - - - - - - -    .0000
         15 - Distance (Nautical Miles)  - -    25.0
         16 - Distance includes all prior
              mission legs beginning with leg     17
         18 - Engine AB (1=ON/0=OFF) - - - -       0
         19 - Maximum TT4 (R)  - - - - - - -    3200.
         ---------------- RESULTS ----------------
         Distance =   25.0 n mi
         CL       =  .0442          CD       = .02837
         Thrust   =  10727.lb       TSFC     = 1.3531 1/hr
         TT4      =   3128.R        Limit    =  Thrust
         Area 0   =   2.47 ft2      Area 9   =   2.68 ft2
         IMS Noz  =  .0338          CD Noz   =  .0056
         Phi I    =  .0743          Phi N    =  .0013
         C        = 1.5184 1/hr     PI       =  .9757
         Beta 17  =  .6846          Beta 18  =  .6680
```

```
MISSION LEG #18  NAME: CLIMB  TYPE: A - CONSTANT SPEED CLIMB
          1 - Initial Altitude (ft)  - - - -   30000.
          2 - Temperature (R)  - - - - - - - -  411.90
              Speed of Sound (A0 - ft/sec) -   994.9
          3 - Velocity (ft/sec)  - - - - - - -  1492.3
          4 - Mach Number  - - - - - - - - - -  1.5000
          5 - Final Altitude (ft)  - - - - -   38000.
         10 - CDR   - - - - - - - - - - - - - -   .0000
         18 - Engine AB (1=ON/0=OFF) - - - -        1
         19 - Maximum TT4 (R)  - - - - - - - -   3200.
         20 - Maximum TT7 (R)  - - - - - - - -   3600.
         ---------------- RESULTS ----------------
         Avg Mach = 1.5267          Avg Vel  = 1492.3 ft/sec
         Time     =     7.3 sec     Distance =   1.78 n mi
         Delta ze =   8000 ft
         CL       =  .0501          CD       = .02849
         CD/CL    =  .5693          Thrust   =  21327.lb
         TT4      =   3200.R        Limit    =   None
         TT7      =   3600.R
         Area 0   =   2.69 ft2      Area 9   =   4.50 ft2
         IMS Noz  =  .0033          CD Noz   =  .0006
         Phi I    =  .0258          Phi N    =  .0000
         u        =  .4351          C        = 2.1397 1/hr
         TSFC     = 1.8734 1/hr     PI       =  .9951
         Beta 18  =  .6680          Beta 19  =  .6647

MISSION LEG #19  NAME: CLIMB  TYPE: A - CONSTANT SPEED CLIMB
          1 - Initial Altitude (ft)  - - - -   38000.
          2 - Temperature (R)  - - - - - - - -  390.00
              Speed of Sound (A0 - ft/sec) -   968.1
          3 - Velocity (ft/sec)  - - - - - - -  1452.1
          4 - Mach Number  - - - - - - - - - -  1.5000
          5 - Final Altitude (ft)  - - - - -   45000.
         10 - CDR   - - - - - - - - - - - - - -   .0000
         18 - Engine AB (1=ON/0=OFF) - - - -        1
         19 - Maximum TT4 (R)  - - - - - - - -   3200.
         20 - Maximum TT7 (R)  - - - - - - - -   3600.
         ---------------- RESULTS ----------------
         Avg Mach = 1.5000          Avg Vel  = 1452.1 ft/sec
         Time     =     8.6 sec     Distance =   2.05 n mi
         Delta ze =   7000 ft
         CL       =  .0738          CD       = .02933
         CD/CL    =  .3974          Thrust   =  15550.lb
         TT4      =   3200.R        Limit    =   None
         TT7      =   3600.R
         Area 0   =   2.77 ft2      Area 9   =   4.56 ft2
         IMS Noz  =  .0041          CD Noz   =  .0007
         Phi I    =  .0202          Phi N    =  .0000
         u        =  .4145          C        = 2.1243 1/hr
         TSFC     = 1.8420 1/hr     PI       =  .9958
         Beta 19  =  .6647          Beta 20  =  .6619
```

```
MISSION LEG #20  NAME: SUBCRS TYPE: E - CONSTANT ALTITUDE/SPEED CRUISE
        1 - Altitude (ft)  - - - - - - - - -   45000.
        2 - Temperature (R)  - - - - - - - -   390.00
            Speed of Sound (AO - ft/sec) -    968.1
        3 - Velocity (ft/sec)  - - - - - - -   871.2
        4 - Mach Number  - - - - - - - - - -   .9000
       10 - CDR   - - - - - - - - - - - - -   .0000
       15 - Distance (Nautical Miles)  - -   100.0
       16 - Distance includes all prior
            mission legs beginning with leg     20
       18 - Engine AB (1=ON/0=OFF) - - - -       0
       19 - Maximum TT4 (R)  - - - - - - - -    3200.
       ---------------- RESULTS ----------------
       Distance =  100.0 n mi
       CL       =  .2415          CD       = .02633
       Thrust   =   1761.1b       TSFC     = 1.3580 1/hr
       TT4      =   2249.R        Limit    =  Thrust
       Area 0*  =   1.88 ft2      Area 9   =   1.63 ft2
       IMS Noz  =  .1321          CD Noz   =  .0170
       Phi I    =  .1620          Phi N    =  .0113
       C        = 1.5661 1/hr     PI       =  .9717
       Beta 20  =  .6619          Beta 21  =  .6432

MISSION LEG #21  NAME: LOITER TYPE: H - LOITER
        1 - Altitude (ft)  - - - - - - - - -   10000.
        2 - Temperature (R)  - - - - - - - -  483.00
            Speed of Sound (AO - ft/sec) -   1077.3
       10 - CDR   - - - - - - - - - - - - -   .0000
       16 - Time (sec)   - - - - - - - - - -  1200.0
       18 - Engine AB (1=ON/0=OFF) - - - -       0
       19 - Maximum TT4 (R)  - - - - - - - -    3200.
       ---------------- RESULTS ----------------
       Mach     =  .4061          Vel      =  437.5 ft/sec
       CL       =  .2449          CD       = .02400
       Thrust   =   1538.1b       TSFC     = 1.6059 1/hr
       TT4      =   1637.R        Limit    =  Thrust
       Area 0*  =   1.16 ft2      Area 9   =   1.27 ft2
       IMS Noz  =  .1915          CD Noz   =  .0007
       Phi I    =  .1370          Phi N    =  .0003
       C        = 1.6642 1/hr     PI       =  .9489
       Beta 21  =  .6432          Beta 22  =  .6103
```

MISSION ANALYSIS CALCULATIONS

MISSION DATA FILE: MISSION5.DAT

MODELS: AIRCRAFT # 5 THRUST # 7 TSFC # 7

ENGINE DATA FILE: AAFSP100.DAT

NUMBER OF ENGINES	2
MAX PRESSURE AT STATION 3	350.0 psia
MAX TEMPERATURE AT STATION 3	1760.0 R
MAX % REF RPM - LP SPOOL	110.0 %
MAX % REF RPM - HP SPOOL	110.0 %

Area 1 = 3.29 Area 10 = 4.03 N Lngth = 4.09

CURRENT SELECTION:

WING LOADING (W/S)	64.00 psf
THRUST LOADING (T/W)	1.20
WEIGHT FRACTION (BETA)	1.0000
AIRCRAFT GROSS TAKEOFF WEIGHT	24400.1b

0

LEG	NAME	PI	BETA I	BETA F	WEIGHT CHANGE	DRAG
1	WARMUP	.988474	1.000000	.988474	281.2 lb	0. lb
2	TOACC	.995504	.988474	.984029	108.4 lb	8124. lb
3	TOROT	.998249	.984029	.982307	42.0 lb	0. lb
4	ACC	.993326	.982307	.975751	160.0 lb	2350. lb
5	C&A1	.994617	.975751	.970499	128.2 lb	3696. lb
6	C&A2	.996617	.970499	.967216	80.1 lb	3699. lb
7	C&A3	.997122	.967216	.964432	67.9 lb	3287. lb
8	C&A4	.996520	.964432	.961075	81.9 lb	2953. lb
9	SUBCRS	.951078	.961075	.914058	1147.2 lb	2804. lb
10	PATROL	.956477	.914058	.874275	970.7 lb	2185. lb
11	PENACC	.986062	.874275	.862090	297.3 lb	5861. lb
12	PEN	.924222	.862090	.796763	1594.0 lb	10846. lb
13	TURN1	.971604	.796763	.774137	552.1 lb	18491. lb
14	TURN2	.971577	.774137	.752134	536.9 lb	15103. lb
15	ACC	.981592	.752134	.738289	337.8 lb	5764. lb
16	DELEXP	1.000000	.738289	.684641	1309.0 lb	0. lb
17	ESC	.975717	.684641	.668016	405.7 lb	10727. lb
18	CLIMB	.995074	.668016	.664725	80.3 lb	9279. lb
19	CLIMB	.995796	.664725	.661931	68.2 lb	6445. lb
20	SUBCRS	.971706	.661931	.643202	457.0 lb	1761. lb
21	LOITER	.948903	.643202	.610336	801.9 lb	1538. lb

SUMMARY OF MISSION WEIGHTS

AIRCRAFT TAKEOFF WEIGHT	24400. lb
WEIGHT OF FUEL USED	8199. lb
WEIGHT OF PAYLOAD EXPENDED	1309. lb
AIRCRAFT LANDING WEIGHT	14892. lb

Appendix E: AAF Engine ONX Design Data

This appendix contains the design data used for creating the solution surfaces and trend analyses. The combinations of π_c and α which fit the requirement that M_5 and M_5' lie between 0.4 and 0.6 were taken from this data.

TURBOFAN ENGINE WITH MIXED EXHAUST

```
********************        INPUT DATA      *********************
     MACH NO    =  1.600                  ALPHA        =   .400
     ALT (FT)   = 35000.                  PI C'        =  3.700
     TO (R)     = 394.10                  PI D (MAX) =    .97
     PO (PSIA) = 3.4680                   PI B         =    .97
     DENSITY    =.00073824                PI N         =    .98
     (SLUG/CUFT)                        EFFICIENCY
     CP C =  .238 BTU/LBM-R               BURNER       =    .98
     CP T =  .295 BTU/LBM-R               MECH HI PR =    .98
     GAMMA C = 1.400                      MECH LO PR =    .99
     GAMMA T = 1.300                      LP COMPR (FAN) =    .89 (EC')
     TT4 MAX = 3200. (R)                  HP COMPR       =    .90 (ECH)
     H - FUEL (BTU/LBM) = 18000.          HP TURBINE     =    .89 (ETH)
     CTO LOW            = .0160           LP TURBINE     =    .91 (ETL)
     CTO HIGH           = .0000           PWR MECH EFF L =    .98
     COOLING AIR #1  =  5.00 %            PWR MECH EFF H =    .98
     COOLING AIR #2  =  5.00 %            BLEED AIR      =  1.00 %
     PO/P9 =  1.00
     *** DRY AFTERBURNER ***              PI AB        =   .96
     *** MIXER ***                        PI MIXER MAX =   .97
************************      RESULTS     *************************
     TAU R =  1.512                       AO =  973.1 FT/SEC
     PI R  =  4.250                       VO = 1556.9 FT/SEC
     PI D  =   .933                       TAU L =  10.064
```

PI C	F/MDOT LBF/LBM/S	S 1/H	M5	M5P	TAUTL	M6	PT9/P9	V9/VO	T EFF	P EFF
10.00	64.42	1.3789	.400	.638	.8698	.448	11.496	2.29	47.65	61.33
11.00	64.24	1.3634	.400	.599	.8684	.445	11.816	2.29	48.16	61.38
12.00	64.00	1.3502	.400	.565	.8670	.442	12.087	2.28	48.58	61.44
13.00	63.73	1.3386	.400	.535	.8657	.439	12.317	2.28	48.94	61.53
14.00	63.42	1.3285	.400	.509	.8644	.436	12.510	2.27	49.24	61.62
15.00	63.10	1.3195	.400	.486	.8632	.433	12.673	2.27	49.49	61.73
16.00	62.75	1.3115	.400	.467	.8620	.430	12.808	2.26	49.70	61.85
17.00	62.40	1.3044	.400	.450	.8609	.427	12.918	2.25	49.88	61.97
18.00	62.03	1.2979	.400	.436	.8597	.425	13.008	2.25	50.02	62.09
19.00	61.65	1.2921	.400	.425	.8586	.422	13.079	2.24	50.15	62.23
20.00	61.27	1.2868	.400	.417	.8575	.421	13.134	2.23	50.25	62.36
21.00	60.88	1.2820	.400	.410	.8564	.419	13.174	2.23	50.33	62.50
22.00	60.50	1.2777	.400	.406	.8554	.418	13.200	2.22	50.39	62.64
23.00	60.10	1.2737	.400	.404	.8544	.417	13.215	2.21	50.44	62.78
24.00	59.71	1.2700	.400	.403	.8533	.417	13.219	2.20	50.47	62.93
25.00	59.32	1.2667	.400	.404	.8523	.417	13.213	2.20	50.49	63.07
26.00	58.92	1.2637	.400	.407	.8514	.417	13.199	2.19	50.49	63.22
27.00	58.52	1.2609	.400	.411	.8504	.418	13.176	2.18	50.49	63.37
28.00	58.13	1.2584	.400	.416	.8494	.419	13.147	2.17	50.48	63.52
29.00	57.73	1.2561	.400	.422	.8485	.420	13.110	2.16	50.45	63.67
30.00	57.34	1.2540	.400	.429	.8475	.421	13.067	2.16	50.42	63.82

TURBOFAN ENGINE WITH MIXED EXHAUST

******************** INPUT DATA *********************

MACH NO = 1.600
ALT (FT) = 35000.
TO (R) = 394.10
PO (PSIA) = 3.4680
DENSITY =.00073824
(SLUG/CUFT)
CP C = .238 BTU/LBM-R
CP T = .295 BTU/LBM-R
GAMMA C = 1.400
GAMMA T = 1.300
TT4 MAX = 3200. (R)
H - FUEL (BTU/LBM) = 18000.
CTO LOW = .0160
CTO HIGH = .0000
COOLING AIR #1 = 5.00 %
COOLING AIR #2 = 5.00 %
PO/P9 = 1.00
*** DRY AFTERBURNER ***
*** MIXER ***

ALPHA = .450
PI C' = 3.700
PI D (MAX) = .97
PI B = .97
PI N = .98
EFFICIENCY
BURNER = .98
MECH HI PR = .98
MECH LO PR = .99
LP COMPR (FAN) = .89 (EC')
HP COMPR = .90 (ECH)
HP TURBINE = .89 (ETH)
LP TURBINE = .91 (ETL)
PWR MECH EFF L = .98
PWR MECH EFF H = .98
BLEED AIR = 1.00 %
PI AB = .96
PI MIXER MAX = .97

************************* RESULTS *************************

TAU R = 1.512
PI R = 4.250
PI D = .933

A0 = 973.1 FT/SEC
V0 = 1556.9 FT/SEC
TAU L = 10.064

PI C	F/MDOT LBF/(LBM/S)	S 1/H	M5	M5P	TAUTL	M6	PT9/P9	V9/V0	T EFF	P EFF
10.00	62.77	1.3663	.400	.668	.8651	.455	11.268	2.26	47.64	61.93
11.00	62.60	1.3509	.400	.631	.8637	.452	11.576	2.26	48.15	61.97
12.00	62.37	1.3378	.400	.599	.8623	.450	11.837	2.25	48.57	62.04
13.00	62.10	1.3263	.400	.571	.8609	.447	12.057	2.25	48.93	62.12
14.00	61.80	1.3163	.400	.547	.8596	.444	12.243	2.24	49.23	62.22
15.00	61.48	1.3075	.400	.526	.8583	.441	12.398	2.24	49.48	62.33
16.00	61.15	1.2996	.400	.509	.8571	.439	12.526	2.23	49.69	62.44
17.00	60.80	1.2925	.400	.494	.8559	.436	12.632	2.22	49.87	62.56
18.00	60.44	1.2862	.400	.482	.8547	.434	12.717	2.22	50.01	62.69
19.00	60.07	1.2805	.400	.472	.8536	.433	12.784	2.21	50.14	62.82
20.00	59.69	1.2753	.400	.464	.8524	.431	12.835	2.20	50.23	62.96
21.00	59.32	1.2706	.400	.459	.8513	.430	12.871	2.19	50.31	63.10
22.00	58.93	1.2663	.400	.456	.8502	.429	12.895	2.19	50.38	63.24
23.00	58.55	1.2624	.400	.454	.8492	.429	12.907	2.18	50.42	63.38
24.00	58.16	1.2588	.400	.454	.8481	.429	12.909	2.17	50.45	63.52
25.00	57.78	1.2556	.400	.455	.8471	.429	12.901	2.17	50.47	63.67
26.00	57.39	1.2527	.400	.458	.8460	.429	12.885	2.16	50.47	63.82
27.00	57.00	1.2500	.400	.462	.8450	.429	12.861	2.15	50.47	63.97
28.00	56.61	1.2476	.400	.467	.8440	.430	12.830	2.14	50.45	64.12
29.00	56.22	1.2454	.400	.473	.8430	.431	12.793	2.14	50.42	64.27
30.00	55.83	1.2434	.400	.480	.8421	.432	12.749	2.13	50.39	64.42

TURBOFAN ENGINE WITH MIXED EXHAUST

```
********************        INPUT DATA      *********************
     MACH NO    -  1.600                    ALPHA        -   .500
     ALT (FT)   - 35000.                    PI C'        -  3.700
     TO (R)     - 394.10                    PI D (MAX) -     .97
     PO (PSIA) - 3.4680                     PI B         -    .97
     DENSITY    -.00073824                  PI N         -    .98
     (SLUG/CUFT)                          EFFICIENCY
     CP C -  .238 BTU/LBM-R                 BURNER       -    .98
     CP T -  .295 BTU/LBM-R                 MECH HI PR -      .98
     GAMMA C - 1.400                        MECH LO PR -      .99
     GAMMA T - 1.300                        LP COMPR (FAN) -     .89 (EC')
     TT4 MAX - 3200. (R)                    HP COMPR         -   .90 (ECH)
     H - FUEL (BTU/LBM) - 18000.            HP TURBINE       -   .89 (ETH)
     CTO LOW          - .0160               LP TURBINE       -   .91 (ETL)
     CTO HIGH         - .0000               PWR MECH EFF L -     .98
     COOLING AIR #1  -  5.00 %              PWR MECH EFF H -     .98
     COOLING AIR #2  -  5.00 %              BLEED AIR        -  1.00 %
     PO/P9 -  1.00
     *** DRY AFTERBURNER ***                PI AB        -    .96
     *** MIXER ***                          PI MIXER MAX -   .97
************************     RESULTS     *************************
     TAU R -  1.512                         AO -  973.1 FT/SEC
     PI R  -  4.250                         VO - 1556.9 FT/SEC
     PI D  -   .933                         TAU L -  10.064
```

PI C	F/MDOT LBF / LBM/S	S 1/H	M5	M5P	TAUTL	M6	PT9/P9	V9/V0	T EFF	P EFF
10.00	61.21	1.3546	.400	.697	.8605	.462	11.048	2.23	47.62	62.51
11.00	61.03	1.3393	.400	.662	.8590	.459	11.346	2.23	48.13	62.55
12.00	60.81	1.3263	.400	.632	.8575	.457	11.597	2.22	48.55	62.62
13.00	60.55	1.3150	.400	.606	.8561	.454	11.808	2.22	48.91	62.70
14.00	60.26	1.3051	.400	.583	.8548	.452	11.986	2.21	49.21	62.80
15.00	59.94	1.2963	.400	.564	.8534	.449	12.134	2.21	49.46	62.91
16.00	59.62	1.2885	.400	.548	.8522	.447	12.257	2.20	49.67	63.02
17.00	59.27	1.2816	.400	.534	.8509	.445	12.357	2.19	49.84	63.14
18.00	58.92	1.2753	.400	.523	.8497	.444	12.438	2.19	49.99	63.27
19.00	58.56	1.2697	.400	.515	.8485	.442	12.500	2.18	50.11	63.40
20.00	58.19	1.2646	.400	.508	.8473	.441	12.548	2.17	50.21	63.54
21.00	57.82	1.2600	.400	.504	.8462	.440	12.581	2.17	50.29	63.68
22.00	57.45	1.2558	.400	.501	.8451	.439	12.602	2.16	50.35	63.82
23.00	57.07	1.2520	.400	.500	.8440	.439	12.612	2.15	50.39	63.96
24.00	56.69	1.2485	.400	.500	.8429	.439	12.612	2.14	50.42	64.11
25.00	56.31	1.2454	.400	.502	.8418	.439	12.602	2.14	50.44	64.25
26.00	55.93	1.2425	.400	.505	.8407	.439	12.585	2.13	50.44	64.40
27.00	55.55	1.2399	.400	.508	.8397	.440	12.559	2.12	50.43	64.55
28.00	55.16	1.2376	.400	.513	.8387	.440	12.527	2.11	50.41	64.70
29.00	54.78	1.2355	.400	.519	.8376	.441	12.489	2.11	50.38	64.85
30.00	54.40	1.2336	.400	.526	.8366	.442	12.444	2.10	50.35	65.01

TURBOFAN ENGINE WITH MIXED EXHAUST

******************** INPUT DATA *********************

MACH NO = 1.600	ALPHA = .550
ALT (FT) = 35000.	PI C' = 3.700
TO (R) = 394.10	PI D (MAX) = .97
PO (PSIA) = 3.4680	PI B = .97
DENSITY =.00073824	PI N = .98
(SLUG/CUFT)	EFFICIENCY
CP C = .238 BTU/LBM-R	BURNER = .98
CP T = .295 BTU/LBM-R	MECH HI PR = .98
GAMMA C = 1.400	MECH LO PR = .99
GAMMA T = 1.300	LP COMPR (FAN) = .89 (EC')
TT4 MAX = 3200. (R)	HP COMPR = .90 (ECH)
H - FUEL (BTU/LBM) = 18000.	HP TURBINE = .89 (ETH)
CTO LOW = .0160	LP TURBINE = .91 (ETL)
CTO HIGH = .0000	PWR MECH EFF L = .98
COOLING AIR #1 = 5.00 %	PWR MECH EFF H = .98
COOLING AIR #2 = 5.00 %	BLEED AIR = 1.00 %
PO/P9 = 1.00	
*** DRY AFTERBURNER ***	PI AB = .96
*** MIXER ***	PI MIXER MAX = .97

************************* RESULTS *************************

TAU R = 1.512	A0 = 973.1 FT/SEC
PI R = 4.250	V0 = 1556.9 FT/SEC
PI D = .933	TAU L = 10.064

PI C	F/MDOT LBF / LBM/S	S 1/H	M5	M5P	TAUTL	M6	PT9/P9	V9/V0	T EFF	P EFF
10.00	59.71	1.3437	.400	.726	.8558	.468	10.837	2.20	47.59	63.07
11.00	59.54	1.3286	.400	.692	.8543	.466	11.124	2.20	48.10	63.12
12.00	59.32	1.3156	.400	.664	.8528	.464	11.365	2.19	48.52	63.18
13.00	59.07	1.3044	.400	.639	.8513	.461	11.569	2.19	48.88	63.27
14.00	58.78	1.2946	.400	.618	.8499	.459	11.739	2.18	49.17	63.36
15.00	58.48	1.2859	.400	.600	.8486	.457	11.881	2.18	49.43	63.47
16.00	58.16	1.2783	.400	.585	.8472	.455	11.998	2.17	49.64	63.59
17.00	57.82	1.2714	.400	.573	.8459	.454	12.093	2.16	49.81	63.71
18.00	57.47	1.2652	.400	.563	.8447	.452	12.169	2.16	49.96	63.84
19.00	57.12	1.2597	.400	.555	.8435	.451	12.229	2.15	50.08	63.97
20.00	56.76	1.2547	.400	.549	.8423	.450	12.272	2.14	50.17	64.10
21.00	56.39	1.2501	.400	.545	.8411	.449	12.303	2.14	50.25	64.24
22.00	56.03	1.2461	.400	.543	.8399	.449	12.321	2.13	50.31	64.38
23.00	55.66	1.2423	.400	.542	.8388	.448	12.329	2.12	50.35	64.53
24.00	55.28	1.2390	.400	.543	.8376	.448	12.327	2.12	50.38	64.67
25.00	54.91	1.2359	.400	.545	.8365	.448	12.315	2.11	50.39	64.82
26.00	54.53	1.2332	.400	.548	.8354	.449	12.296	2.10	50.39	64.97
27.00	54.16	1.2307	.400	.552	.8343	.449	12.270	2.09	50.38	65.12
28.00	53.78	1.2285	.400	.557	.8333	.449	12.237	2.09	50.36	65.27
29.00	53.40	1.2265	.400	.562	.8322	.450	12.197	2.08	50.33	65.42
30.00	53.03	1.2247	.400	.569	.8312	.451	12.152	2.07	50.29	65.58

TURBOFAN ENGINE WITH MIXED EXHAUST

```
********************        INPUT DATA      *********************
    MACH NO   =  1.600                  ALPHA        =   .600
    ALT (FT)  = 35000.                  PI C'        =  3.700
    TO (R)    = 394.10                  PI D (MAX) =    .97
    PO (PSIA) = 3.4680                  PI B         =    .97
    DENSITY   =.00073824                PI N         =    .98
    (SLUG/CUFT)                       EFFICIENCY
    CP C =  .238 BTU/LBM-R              BURNER       =    .98
    CP T =  .295 BTU/LBM-R              MECH HI PR =    .98
    GAMMA C = 1.400                     MECH LO PR =    .99
    GAMMA T = 1.300                     LP COMPR (FAN) =   .89 (EC')
    TT4 MAX = 3200. (R)                 HP COMPR       =   .90 (ECH)
    H - FUEL (BTU/LBM) = 18000.         HP TURBINE     =   .89 (ETH)
    CTO LOW          = .0160            LP TURBINE     =   .91 (ETL)
    CTO HIGH         = .0000            PWR MECH EFF L =   .98
    COOLING AIR #1   =   5.00 %         PWR MECH EFF H =   .98
    COOLING AIR #2   =   5.00 %         BLEED AIR      =  1.00 %
    PO/P9 =   1.00
    *** DRY AFTERBURNER ***             PI AB        =   .96
    *** MIXER ***                       PI MIXER MAX =    .97
*************************      RESULTS    *************************
    TAU R =  1.512                      AO =   973.1 FT/SEC
    PI R  =  4.250                      VO = 1556.9 FT/SEC
    PI D  =   .933                      TAU L =  10.064
```

PI C	F/MDOT LBF / LBM/S	S 1/H	M5	M5P	TAUTL	M6	PT9/P9	V9/VO	T EFF	P EFF
10.00	58.28	1.3336	.400	.754	.8512	.474	10.633	2.17	47.55	63.62
11.00	58.12	1.3186	.400	.722	.8496	.472	10.910	2.17	48.05	63.67
12.00	57.91	1.3058	.400	.694	.8480	.470	11.143	2.16	48.48	63.73
13.00	57.66	1.2946	.400	.671	.8465	.468	11.338	2.16	48.83	63.82
14.00	57.38	1.2849	.400	.651	.8451	.466	11.502	2.16	49.13	63.91
15.00	57.08	1.2763	.400	.634	.8437	.464	11.638	2.15	49.38	64.02
16.00	56.76	1.2687	.400	.620	.8423	.463	11.749	2.14	49.59	64.14
17.00	56.43	1.2620	.400	.609	.8410	.461	11.840	2.14	49.77	64.26
18.00	56.09	1.2559	.400	.600	.8397	.460	11.912	2.13	49.91	64.39
19.00	55.74	1.2504	.400	.593	.8384	.459	11.967	2.12	50.03	64.52
20.00	55.39	1.2455	.400	.588	.8372	.458	12.008	2.12	50.13	64.65
21.00	55.03	1.2411	.400	.584	.8359	.458	12.035	2.11	50.20	64.79
22.00	54.67	1.2371	.400	.583	.8347	.457	12.051	2.10	50.26	64.93
23.00	54.30	1.2335	.400	.582	.8336	.457	12.057	2.10	50.30	65.08
24.00	53.94	1.2302	.400	.583	.8324	.457	12.052	2.09	50.32	65.22
25.00	53.57	1.2272	.400	.585	.8312	.457	12.040	2.08	50.33	65.37
26.00	53.20	1.2246	.400	.588	.8301	.457	12.019	2.07	50.33	65.52
27.00	52.83	1.2222	.400	.592	.8290	.458	11.991	2.07	50.32	65.67
28.00	52.46	1.2200	.400	.597	.8279	.458	11.957	2.06	50.30	65.82
29.00	52.09	1.2181	.400	.603	.8268	.459	11.917	2.05	50.26	65.98
30.00	51.72	1.2165	.400	.609	.8257	.459	11.872	2.05	50.22	66.13

TURBOFAN ENGINE WITH MIXED EXHAUST

```
********************      INPUT DATA      **********************
    MACH NO    = 1.600                 ALPHA        =  .650
    ALT (FT)   = 35000.                PI C'        = 3.700
    TO (R)     = 394.10                PI D (MAX) =   .97
    PO (PSIA) = 3.4680                 PI B         =   .97
    DENSITY    =.00073824              PI N         =   .98
    (SLUG/CUFT)                      EFFICIENCY
    CP C =  .238 BTU/LBM-R              BURNER       =   .98
    CP T =  .295 BTU/LBM-R              MECH HI PR =   .98
    GAMMA C = 1.400                     MECH LO PR =   .99
    GAMMA T = 1.300                     LP COMPR (FAN) =   .89 (EC')
    TT4 MAX = 3200. (R)                 HP COMPR       =   .90 (ECH)
    H - FUEL (BTU/LBM) = 18000.         HP TURBINE     =   .89 (ETH)
    CTO LOW          = .0160            LP TURBINE     =   .91 (ETL)
    CTO HIGH         = .0000            PWR MECH EFF L =   .98
    COOLING AIR #1  =  5.00 %           PWR MECH EFF H =   .98
    COOLING AIR #2  =  5.00 %           BLEED AIR      =  1.00 %
    PO/P9 =  1.00
    *** DRY AFTERBURNER ***             PI AB        =   .96
    *** MIXER ***                       PI MIXER MAX =    .97
*************************     RESULTS     *************************
    TAU R =  1.512                      AO =  973.1 FT/SEC
    PI R  =  4.250                      VO = 1556.9 FT/SEC
    PI D  =   .933                      TAU L = 1[illegible].[illegible]04
```

PI C	F/MDOT LBF/LBM/S	S 1/H	M5	M5P	TAUTL	M6	PT9/P9	V9/VO	T EFF	P EFF
10.00	56.91	1.3243	.400	.781	.8465	.480	10.436	2.14	47.49	64.16
11.00	56.75	1.3094	.400	.750	.8449	.478	10.703	2.14	48.00	64.21
12.00	56.55	1.2966	.400	.724	.8433	.477	10.928	2.14	48.43	64.27
13.00	56.30	1.2856	.400	.702	.8417	.475	11.116	2.13	48.78	64.35
14.00	56.03	1.2759	.400	.683	.8402	.473	11.273	2.13	49.08	64.45
15.00	55.74	1.2675	.400	.667	.8388	.472	11.403	2.12	49.33	64.56
16.00	55.42	1.2599	.400	.654	.8374	.470	11.509	2.12	49.54	64.67
17.00	55.10	1.2532	.400	.644	.8360	.469	11.595	2.11	49.71	64.80
18.00	54.77	1.2473	.400	.635	.8347	.468	11.663	2.10	49.86	64.92
19.00	54.43	1.2419	.400	.629	.8334	.467	11.715	2.10	49.97	65.05
20.00	54.08	1.2371	.400	.625	.8321	.466	11.753	2.09	50.07	65.19
21.00	53.73	1.2327	.400	.622	.8308	.466	11.778	2.08	50.14	65.33
22.00	53.37	1.2288	.400	.620	.8296	.465	11.791	2.08	50.20	65.47
23.00	53.01	1.2253	.400	.620	.8284	.465	11.794	2.07	50.23	65.62
24.00	52.65	1.2221	.400	.621	.8272	.465	11.788	2.06	50.26	65.76
25.00	52.29	1.2192	.400	.624	.8260	.465	11.774	2.06	50.27	65.91
26.00	51.92	1.2167	.400	.627	.8248	.465	11.752	2.05	50.26	66.06
27.00	51.56	1.2144	.400	.631	.8237	.466	11.723	2.04	50.25	66.21
28.00	51.19	1.2123	.400	.636	.8225	.466	11.688	2.04	50.22	66.36
29.00	50.83	1.2105	.400	.642	.8214	.466	11.648	2.03	50.18	66.52
30.00	50.46	1.2089	.400	.648	.8203	.467	11.602	2.02	50.14	66.67

Appendix F: SAS Modeling Results for the Two-Variable Design Example

This appendix contains the SAS results used for determining the best design choices for the two-variable analysis example.

```
TITLE 'MODELING MISSION FUEL CONSUMPTION AND THRUST AT SEA LEVEL';
OPTIONS LINESIZE=78;
DATA ENGINES;
  INPUT FUEL THRUST MACH ALT PIC ALPHA PICP MDOT;
  PICSQ=PIC**2;
  MACHSQ=MACH**2;
  ALTSQ=ALT**2;
  ALPHASQ=ALPHA**2;
  PICPSQ=PICP**2;
  MDOTSQ=MDOT**2;
  B=PIC*ALPHA;
  D=PIC**2*ALPHA**2;
CARDS;
8579 14539 1.6 35 15 .4 3.7 100
8410 14400 1.6 35 16 .5 3.7 100
8314 14465 1.6 35 18 .45 3.7 100
8268 14375 1.6 35 18 .55 3.7 100
8243 14394 1.6 35 20 .5 3.7 100
8159 14166 1.6 35 20 .6 3.7 100
8143 13079 1.6 35 22 .55 3.7 100
. . 1.6 35 22 .65 3.7 100
. . 1.6 35 22 .45 3.7 100
. . 1.6 35 21 .6 3.7 100
. . 1.6 35 21 .55 3.7 100
. . 1.6 35 21 .4 3.7 100
. . 1.6 35 20 .65 3.7 100
. . 1.6 35 20 .55 3.7 100
. . 1.6 35 20 .45 3.7 100
. . 1.6 35 19 .6 3.7 100
. . 1.6 35 19 .45 3.7 100
. . 1.6 35 17.5 .5 3.7 100
. . 1.6 35 17 .6 3.7 100
. . 1.6 35 17 .4 3.7 100
. . 1.6 35 16 .6 3.7 100
. . 1.6 35 16 .45 3.7 100
. . 1.6 35 15 .6 3.7 100
;
PROC PRINT;
PROC REG SIMPLE;
  MODEL THRUST=PIC ALPHA D/CLI NOINT;
  MODEL THRUST=PIC ALPHA PICSQ B/CLI NOINT;
  MODEL THRUST=PIC ALPHA ALPHASQ PICSQ/CLI NOINT;
  MODEL FUEL=PIC ALPHA PICSQ B/CLI;
  MODEL FUEL=PIC ALPHA ALPHASQ PICSQ/CLI;
  MODEL FUEL=PIC ALPHA PICSQ B D/CLI;
```

OBS	FUEL	THRUST	MACH	ALT	PIC	ALPHA	PICP	MDOT	PICSQ
1	8579	14539	1.6	35	15.0	0.40	3.7	100	225.00
2	8410	14400	1.6	35	16.0	0.50	3.7	100	256.00
3	8314	14465	1.6	35	18.0	0.45	3.7	100	324.00
4	8268	14375	1.6	35	18.0	0.55	3.7	100	324.00
5	8243	14394	1.6	35	20.0	0.50	3.7	100	400.00
6	8159	14166	1.6	35	20.0	0.60	3.7	100	400.00
7	8143	13079	1.6	35	22.0	0.55	3.7	100	484.00
8	.	.	1.6	35	22.0	0.65	3.7	100	484.00
9	.	.	1.6	35	22.0	0.45	3.7	100	484.00
10	.	.	1.6	35	21.0	0.60	3.7	100	441.00
11	.	.	1.6	35	21.0	0.55	3.7	100	441.00
12	.	.	1.6	35	21.0	0.40	3.7	100	441.00
13	.	.	1.6	35	20.0	0.65	3.7	100	400.00
14	.	.	1.6	35	20.0	0.55	3.7	100	400.00
15	.	.	1.6	35	20.0	0.45	3.7	100	400.00
16	.	.	1.6	35	19.0	0.60	3.7	100	361.00
17	.	.	1.6	35	19.0	0.45	3.7	100	361.00
18	.	.	1.6	35	17.5	0.50	3.7	100	306.25
19	.	.	1.6	35	17.0	0.60	3.7	100	289.00
20	.	.	1.6	35	17.0	0.40	3.7	100	289.00
21	.	.	1.6	35	16.0	0.60	3.7	100	256.00
22	.	.	1.6	35	16.0	0.45	3.7	100	256.00
23	.	.	1.6	35	15.0	0.60	3.7	100	225.00

OBS	MACHSQ	ALTSQ	ALPHASQ	PICPSQ	MDOTSQ	B	D
1	2.56	1225	0.1600	13.69	10000	6.00	36.000
2	2.56	1225	0.2500	13.69	10000	8.00	64.000
3	2.56	1225	0.2025	13.69	10000	8.10	65.610
4	2.56	1225	0.3025	13.69	10000	9.90	98.010
5	2.56	1225	0.2500	13.69	10000	10.00	100.000
6	2.56	1225	0.3600	13.69	10000	12.00	144.000
7	2.56	1225	0.3025	13.69	10000	12.10	146.410
8	2.56	1225	0.4225	13.69	10000	14.30	204.490
9	2.56	1225	0.2025	13.69	10000	9.90	98.010
10	2.56	1225	0.3600	13.69	10000	12.60	158.760
11	2.56	1225	0.3025	13.69	10000	11.55	133.403
12	2.56	1225	0.1600	13.69	10000	8.40	70.560
13	2.56	1225	0.4225	13.69	10000	13.00	169.000
14	2.56	1225	0.3025	13.69	10000	11.00	121.000
15	2.56	1225	0.2025	13.69	10000	9.00	81.000
16	2.56	1225	0.3600	13.69	10000	11.40	129.960
17	2.56	1225	0.2025	13.69	10000	8.55	73.103
18	2.56	1225	0.2500	13.69	10000	8.75	76.563
19	2.56	1225	0.3600	13.69	10000	10.20	104.040
20	2.56	1225	0.1600	13.69	10000	6.80	46.240
21	2.56	1225	0.3600	13.69	10000	9.60	92.160
22	2.56	1225	0.2025	13.69	10000	7.20	51.840
23	2.56	1225	0.3600	13.69	10000	9.00	81.000

Descriptive Statistics

Variables	Sum	Mean	Uncorrected SS
INTERCEP	7	1	7
PIC	129	18.428571429	2413
ALPHA	3.55	0.5071428571	1.8275
D	654.03	93.432857143	71474.5203
THRUST	99418	14202.571429	1413542404
PICSQ	2413	344.71428571	880369
B	66.1	9.4428571429	654.03
ALPHASQ	1.8275	0.2610714286	0.50421875
FUEL	58116	8302.2857143	482634540

Variables	Variance	Std Deviation
INTERCEP	0	0
PIC	5.9523809524	2.4397501824
ALPHA	0.0045238095	0.0672592709
D	1727.7714571	41.566470347
THRUST	258526.28571	508.45480204
PICSQ	8095.5714286	89.975393462
B	4.9761904762	2.2307376529
ALPHASQ	0.0045184524	0.0672194345
FUEL	23150.571429	152.15311837

Model: MODEL1
NOTE: No intercept in model. R-square is redefined.
Dependent Variable: THRUST

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	3	1412616465.6	470872155.22	2034.140	0.0001
Error	4	925938.35499	231484.58875		
U Total	7	1413542404			

Root MSE	481.12845	R-square	0.9993
Dep Mean	14202.57143	Adj R-sq	0.9989
C.V.	3.38762		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
PIC	1	498.701031	104.07926820	4.792	0.0087
ALPHA	1	22357	3746.6443856	5.967	0.0040
D	1	-67.769722	6.50490755	-10.418	0.0005

Obs	Dep Var THRUST	Predict Value	Std Err Predict	Lower95% Predict	Upper95% Predict	Residual
1	14539.0	13983.4	310.175	12394.1	15572.8	555.6
2	14400.0	14820.2	319.913	13216.1	16424.4	-420.2
3	14465.0	14590.7	288.515	13033.1	16148.3	-125.7
4	14375.0	14630.6	275.186	13091.7	16169.5	-255.6
5	14394.0	14375.3	264.498	12851.0	15899.7	18.6928
6	14166.0	13629.1	358.870	11962.6	15295.6	536.9
7	13079.0	13345.3	371.734	11657.3	15033.4	-266.3
8	.	11644.9	623.529	9458.3	13831.6	.
9	.	14389.7	607.029	12239.2	16540.3	.
10	.	13127.5	380.117	11425.1	14829.9	.
11	.	13728.2	280.929	12181.3	15275.0	.
12	.	14633.5	701.960	12270.7	16996.3	.
13	.	13052.7	562.952	10996.6	15108.7	.
14	.	14070.0	222.927	12597.7	15542.2	.
15	.	14545.1	425.333	12762.2	16328.1	.
16	.	14081.9	383.379	12373.8	15789.9	.
17	.	14581.6	347.979	12933.0	16230.2	.
18	.	14716.9	216.269	13252.4	16181.5	.
19	.	14841.1	526.454	12861.0	16821.2	.
20	.	14286.9	382.705	12580.0	15993.7	.
21	.	15147.5	620.903	12966.6	17328.3	.
22	.	14526.5	267.731	12997.8	16055.2	.
23	.	15405.1	721.492	12997.4	17812.8	.

```
Sum of Residuals                  43.36123
Sum of Squared Residuals       925938.3550
Predicted Resid SS (Press)    3556090.1815
```

Model: MODEL2
NOTE: No intercept in model. R-square is redefined.
Dependent Variable: THRUST

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	4	1413417256.6	353354314.14	8470.513	0.0001
Error	3	125147.43104	41715.81035		
U Total	7	1413542404			

Root MSE	204.24449	R-square	0.9999
Dep Mean	14202.57143	Adj R-sq	0.9998
C.V.	1.43808		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
PIC	1	2221.176536	451.28504867	4.922	0.0161
ALPHA	1	-17745	16601.933980	-1.069	0.3635
PICSQ	1	-75.604434	23.93552439	-3.159	0.0509
B	1	882.244408	888.27395877	0.993	0.3938

Obs	Dep Var THRUST	Predict Value	Std Err Predict	Lower95% Predict	Upper95% Predict	Residual
1	14539.0	14501.9	152.187	13691.3	15312.5	37.0716
2	14400.0	14369.3	183.465	13495.6	15243.0	30.6909
3	14465.0	14646.1	133.104	13870.2	15421.9	-181.1
4	14375.0	14459.6	116.944	13710.5	15208.6	-84.5515
5	14394.0	14131.5	112.686	13389.1	14873.8	262.5
6	14166.0	14121.4	185.689	13242.9	14999.9	44.5927
7	13079.0	13188.5	177.404	12327.5	14049.5	-109.5
8	.	13354.9	307.202	12180.8	14528.9	.
9	.	13022.1	461.415	11416.2	14628.0	.
10	.	13772.1	186.891	12891.1	14653.2	.
11	.	13733.1	119.486	12980.0	14486.1	.
12	.	13615.8	425.354	12114.2	15117.5	.
13	.	14116.4	282.636	13006.6	15226.1	.
14	.	14126.4	109.933	13388.3	14864.6	.
15	.	14136.5	190.575	13247.5	15025.5	.
16	.	14319.5	174.783	13463.9	15175.0	.
17	.	14466.9	146.357	13667.2	15266.5	.
18	.	14563.6	88.706	13855.0	15272.3	.
19	.	14261.9	281.128	13156.0	15367.8	.
20	.	14811.4	206.379	13887.3	15735.5	.
21	.	14006.4	446.341	12444.2	15568.5	.
22	.	14550.8	114.846	13805.1	15296.5	.
23	.	13599.6	671.916	11364.6	15834.5	.

Sum of Residuals	-0.20635
Sum of Squared Residuals	125147.4310
Predicted Resid SS (Press)	554407.0485

Model: MODEL3
NOTE: No intercept in model. R-square is redefined.
Dependent Variable: THRUST

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	4	1413414284	353353571	8273.967	0.0001
Error	3	128120.00466	42706.66822		
U Total	7	1413542404			

Root MSE	206.65592	R-square	0.9999
Dep Mean	14202.57143	Adj R-sq	0.9998
C.V.	1.45506		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
PIC	1	2266.201050	520.67499340	4.352	0.0224
ALPHA	1	-19381	19154.638118	-1.012	0.3861
ALPHASQ	1	17425	18429.262131	0.946	0.4142
PICSQ	1	-64.634572	13.61000100	-4.749	0.0177

Obs	Dep Var THRUST	Predict Value	Std Err Predict	Lower95% Predict	Upper95% Predict	Residual
1	14539.0	14485.8	150.796	13671.6	15299.9	53.2393
2	14400.0	14378.4	182.394	13501.2	15255.6	21.5818
3	14465.0	14657.0	140.714	13861.4	15452.7	-192.0
4	14375.0	14461.4	118.362	13703.5	15219.3	-86.4206
5	14394.0	14135.8	113.857	13384.9	14886.7	258.2
6	14166.0	14114.5	186.141	13229.3	14999.6	51.5289
7	13079.0	13184.7	182.751	12306.7	14062.7	-105.7
8	.	13337.6	303.866	12168.1	14507.1	.
9	.	13380.3	262.675	12316.7	14444.0	.
10	.	13730.7	166.686	12885.7	14575.6	.
11	.	13697.8	122.609	12933.1	14462.5	.
12	.	14121.9	363.302	12791.7	15452.1	.
13	.	14234.5	384.411	12845.5	15623.4	.
14	.	14081.6	96.167	13356.2	14807.0	.
15	.	14277.2	195.278	13372.4	15182.1	.
16	.	14369.0	205.531	13441.4	15296.6	.
17	.	14531.8	170.192	13679.8	15383.8	.
18	.	14529.8	94.339	13806.9	15252.8	.
19	.	14490.3	236.633	13490.5	15490.1	.
20	.	14881.6	260.424	13823.5	15939.6	.
21	.	14357.0	267.308	13281.8	15432.3	.
22	.	145[illegible]9.8	125.044	13751.1	15288.5	.
23	.	1[illegible]094.5	325.949	12866.3	15322.8	.

Sum of Residuals	0.34916
Sum of Squared Residuals	128120.0047
Predicted Resid SS (Press)	614342.9747

Model: MODEL4
Dependent Variable: FUEL

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	4	137787.25036	34446.81259	61.723	0.0160
Error	2	1116.17822	558.08911		
C Total	6	138903.42857			

Root MSE	23.62391	R-square	0.9920
Dep Mean	8302.28571	Adj R-sq	0.9759
C.V.	0.28455		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	11472	705.32403309	16.265	0.0038
PIC	1	-235.758661	77.15070873	-3.056	0.0925
ALPHA	1	-1749.844789	2073.4895355	-0.844	0.4876
PICSQ	1	4.389897	2.90845631	1.509	0.2703
B	1	58.108370	112.43356293	0.517	0.6568

Obs	Dep Var FUEL	Predict Value	Std Err Predict	Lower95% Predict	Upper95% Predict	Residual
1	8579.0	8572.5	22.987	8430.7	8714.3	6.5078
2	8410.0	8414.1	21.260	8277.3	8550.8	-4.0527
3	8314.0	8334.4	18.705	8204.7	8464.0	-20.3514
4	8268.0	8264.0	15.074	8143.4	8384.5	4.0380
5	8243.0	8219.4	15.756	8097.2	8341.6	23.6200
6	8159.0	8160.6	21.894	8022.0	8299.2	-1.6123
7	8143.0	8151.1	22.480	8010.8	8291.5	-8.1494
8	.	8104.0	47.431	7876.0	8332.0	.
9	.	8198.3	54.943	7941.0	8455.6	.
10	.	8139.7	24.431	7993.5	8285.9	.
11	.	8166.2	14.053	8047.9	8284.5	.
12	.	8245.6	54.721	7989.2	8502.1	.
13	.	8131.2	34.430	7951.6	8310.9	.
14	.	8190.0	12.922	8074.1	8305.9	.
15	.	8248.8	26.893	8094.7	8402.8	.
16	.	8190.3	20.246	8056.4	8324.2	.
17	.	8287.2	21.908	8148.5	8425.8	.
18	.	8324.6	12.692	8209.2	8440.0	.
19	.	8276.0	33.151	8100.9	8451.2	.
20	.	8428.4	24.444	8282.1	8574.7	.
21	.	8332.0	52.028	8086.2	8577.9	.
22	.	8455.1	13.346	8338.3	8571.8	.
23	.	8396.8	77.858	8046.8	8746.9	.

Sum of Residuals	9.5E-13
Sum of Squared Residuals	1116.1782
Predicted Resid SS (Press)	27807.6847

Model: MODEL5
Dependent Variable: FUEL

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	4	137757.89584	34439.47396	60.128	0.0164
Error	2	1145.53273	572.76636		
C Total	6	138903.42857			

Root MSE	23.93254	R-square	0.9918
Dep Mean	8302.28571	Adj R-sq	0.9753
C.V.	0.28826		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	11496	753.57667504	15.256	0.0043
PIC	1	-236.101345	78.82521667	-2.995	0.0957
ALPHA	1	-1833.003889	2522.8309458	-0.727	0.5430
ALPHASQ	1	1126.117952	2463.1919266	0.457	0.6924
PICSQ	1	5.200016	2.08497870	2.494	0.1301

Obs	Dep Var FUEL	Predict Value	Std Err Predict	Lower95% Predict	Upper95% Predict	Residual
1	8579.0	8571.8	23.176	8428.5	8715.2	7.1653
2	8410.0	8415.0	21.296	8277.1	8552.8	-4.9841
3	8314.0	8334.5	18.999	8203.1	8466.0	-20.5421
4	8268.0	8263.9	15.373	8141.5	8386.2	4.1465
5	8243.0	8219.4	16.104	8095.3	8343.5	23.6189
6	8159.0	8160.0	22.339	8019.1	8300.8	-0.9537
7	8143.0	8151.5	22.750	8009.4	8293.5	-8.4507
8	.	8103.3	50.162	7864.1	8342.4	.
9	.	8222.1	30.920	8053.9	8390.4	.
10	.	8137.1	22.243	7996.5	8277.6	.
11	.	8164.0	14.205	8044.2	8283.7	.
12	.	8278.4	42.423	8068.9	8488.0	.
13	.	8138.7	49.942	7900.4	8377.0	.
14	.	8186.9	12.114	8071.4	8302.3	.
15	.	8257.5	23.850	8112.2	8402.9	.
16	.	8193.3	24.044	8047.3	8339.2	.
17	.	8290.8	22.118	8150.6	8431.1	.
18	.	8322.1	15.001	8200.6	8443.7	.
19	.	8291.1	28.203	8131.9	8450.2	.
20	.	8432.4	30.178	8266.7	8598.2	.
21	.	8355.6	33.346	8179.0	8532.2	.
22	.	8453.1	14.518	8332.7	8573.6	.
23	.	8430.5	43.125	8218.2	8642.7	.

Sum of Residuals	0
Sum of Squared Residuals	1145.5327
Predicted Resid SS (Press)	26573.8607

Model: MODEL6
Dependent Variable: FUEL

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	5	138594.90499	27718.98100	89.844	0.0799
Error	1	308.52359	308.52359		
C Total	6	138903.42857			

Root MSE	17.56484	R-square	0.9978
Dep Mean	8302.28571	Adj R-sq	0.9867
C.V.	0.21157		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	17739	3908.4586594	4.539	0.1381
PIC	1	-758.240366	327.98085619	-2.312	0.2599
ALPHA	1	-17795	10036.114849	-1.773	0.3269
PICSQ	1	6.451742	2.51004741	2.570	0.2362
B	1	1487.021376	887.10306668	1.676	0.3424
D	1	-28.945486	17.89007601	-1.618	0.3524

Obs	Dep Var FUEL	Predict Value	Std Err Predict	Lower95% Predict	Upper95% Predict	Residual
1	8579.0	8579.0	17.565	8263.4	8894.7	-0.0477
2	8410.0	8404.9	16.797	8096.1	8713.7	5.1377
3	8314.0	8319.0	16.850	8009.7	8628.2	-4.9590
4	8268.0	8278.2	14.267	7990.7	8565.8	-10.2456
5	8243.0	8233.0	14.410	7944.3	8521.6	10.0431
6	8159.0	8153.9	16.802	7845.0	8462.7	5.1198
7	8143.0	8148.0	16.824	7839.0	8457.1	-5.0483
8	.	7958.8	96.411	6713.7	9204.0	.
9	.	8057.1	96.366	6812.5	9301.7	.
10	.	8125.1	20.273	7784.3	8466.0	.
11	.	8187.5	16.821	7878.5	8496.5	.
12	.	7991.7	162.139	5919.5	10063.9	.
13	.	8027.5	69.029	7122.5	8932.5	.
14	.	8222.4	22.193	7862.7	8582.0	.
15	.	8185.7	43.829	7585.7	8785.6	.
16	.	8174.7	17.882	7856.2	8493.2	.
17	.	8251.7	27.301	7839.2	8664.2	.
18	.	8343.3	14.946	8050.3	8636.4	.
19	.	8192.5	57.211	7432.1	8952.9	.
20	.	8368.7	41.143	7800.3	8937.1	.
21	.	8189.5	96.233	6946.5	9432.4	.
22	.	8457.0	9.994	8200.2	8713.8	.
23	.	8178.5	146.828	6299.6	10057.4	.

Sum of Residuals 9.98E-13
Sum of Squared Residuals 308.5236
Predicted Resid SS (Press) 41932612.664

Appendix G: SAS Modeling Results for the Six-Variable Design Example

This appendix contains the SAS results used for determining the best design choices for the six-variable analysis example .

```
TITLE 'MODELING MISSION FUEL CONSUMPTION AND THRUST AT SEA LEVEL';
OPTIONS LINESIZE=78;
DATA ENGINES;
  INPUT FUEL THRUST MACH ALT PIC ALPHA PICP MDOT;
  PICSQ=PIC**2;
  MACHSQ=MACH**2;
  ALTSQ=ALT**2;
  ALPHASQ=ALPHA**2;
  PICPSQ=PICP**2;
  MDOTSQ=MDOT**2;
  B=PIC*ALPHA;
  C=PICP*ALPHA;
  D=PIC**2*ALPHA**2;
CARDS;
8203 14972 1.5 30 20 .5 3.6 115
7784 15029 1.7 35 18 .7 3.6 120
8268 12904 1.5 25 21 .5 3.3 130
8579 14539 1.6 35 15 .4 3.7 100
8118 17004 1.4 30 18 .55 3.5 120
7895 17710 1.6 38 22 .5 3.7 120
8130 14161 1.4 25 22 .6 3.6 130
8393 12820 1.7 30 18 .5 3.4 120
7997 18288 1.4 35 17 .7 3.7 110
8071 12855 1.6 35 22 .65 3.7 100
8292 14509 1.6 35 20 .45 3.7 100
8190 14308 1.6 35 19 .6 3.7 100
8048 16469 1.5 35 19 .6 3.7 105
7911 17653 1.5 40 20 .6 3.7 90
;
PROC PRINT;
PROC REG SIMPLE;
  MODEL FUEL=MACH ALT PIC ALPHA ALPHASQ PICSQ MDOTSQ/NOINT CLI;
  MODEL FUEL=MACH ALT PIC ALPHA ALPHASQ PICSQ PICPSQ/NOINT CLI;
  MODEL FUEL=MACH ALT PIC D ALPHASQ PICSQ MDOTSQ/NOINT CLI;
  MODEL THRUST=MACH ALT PIC ALPHA ALPHASQ PICSQ MDOTSQ/CLI;
  MODEL THRUST=MACH ALT PIC D ALPHASQ PICSQ MDOTSQ/CLI;
```

OBS	FUEL	THRUST	MACH	ALT	PIC	ALPHA	PICP	MDOT	PICSQ	MACHSQ
1	8203	14972	1.5	30	20	0.50	3.6	115	400	2.25
2	7784	15029	1.7	35	18	0.70	3.6	120	324	2.89
3	8268	12904	1.5	25	21	0.50	3.3	130	441	2.25
4	8579	14539	1.6	35	15	0.40	3.7	100	225	2.56
5	8118	17004	1.4	30	18	0.55	3.5	120	324	1.96
6	7895	17710	1.6	38	22	0.50	3.7	120	484	2.56
7	8130	14161	1.4	25	22	0.60	3.6	130	484	1.96
8	8393	12820	1.7	30	18	0.50	3.4	120	324	2.89
9	7997	18288	1.4	35	17	0.70	3.7	110	289	1.96
10	8071	12855	1.6	35	22	0.65	3.7	100	484	2.56
11	8292	14509	1.6	35	20	0.45	3.7	100	400	2.56
12	8190	14308	1.6	35	19	0.60	3.7	100	361	2.56
13	8048	16469	1.5	35	19	0.60	3.7	105	361	2.25
14	7911	17653	1.5	40	20	0.60	3.7	90	400	2.25

OBS	ALTSQ	ALPHASQ	PICPSQ	MDOTSQ	B	C	D
1	900	0.2500	12.96	13225	10.0	1.800	100.00
2	1225	0.4900	12.96	14400	12.6	2.520	158.76
3	625	0.2500	10.89	16900	10.5	1.650	110.25
4	1225	0.1600	13.69	10000	6.0	1.480	36.00
5	900	0.3025	12.25	14400	9.9	1.925	98.01
6	1444	0.2500	13.69	14400	11.0	1.850	121.00
7	625	0.3600	12.96	16900	13.2	2.160	174.24
8	900	0.2500	11.56	14400	9.0	1.700	81.00
9	1225	0.4900	13.69	12100	11.9	2.590	141.61
10	1225	0.4225	13.69	10000	14.3	2.405	204.49
11	1225	0.2025	13.69	10000	9.0	1.665	81.00
12	1225	0.3600	13.69	10000	11.4	2.220	129.96
13	1225	0.3600	13.69	11025	11.4	2.220	129.96
14	1600	0.3600	13.69	8100	12.0	2.220	144.00

Descriptive Statistics

Variables	Sum	Mean	Uncorrected SS
INTERCEP	14	1	14
MACH	21.6	1.5428571429	33.46
ALT	463	33.071428571	15569
PIC	271	19.357142857	5301
ALPHA	7.85	0.5607142857	4.5075
ALPHASQ	4.5075	0.3219642857	1.58521875
PICSQ	5301	378.64285714	2086965
MDOTSQ	175850	12560.714286	2309131250
FUEL	113879	8134.2142857	926891867
PICPSQ	183.1	13.078571429	2405.5018
D	1710.28	122.16285714	232769.0932
THRUST	213221	15230.071429	3293382083

Variables	Variance	Std Deviation
INTERCEP	0	0
MACH	0.0103296703	0.1016349858
ALT	19.763736264	4.4456423904
PIC	4.2472527473	2.0608863984
ALPHA	0.0081456044	0.0902530021
ALPHASQ	0.0103049794	0.1015134444
PICSQ	6136.8626374	78.338130163
MDOTSQ	7717664.8352	2778.068544
FUEL	44282.950549	210.43514571
PICPSQ	0.8319516484	0.9121138352
D	1833.5693758	42.820198223
THRUST	3539309.456	1881.3052533

Model: MODEL1
NOTE: No intercept in model. R-square is redefined.
Dependent Variable: FUEL

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	7	926489467.26	132355638.18	2302.411	0.0001
Error	7	402399.73569	57485.67653		
U Total	14	926891867			

Root MSE	239.76171	R-square	0.9996
Dep Mean	8134.21429	Adj R-sq	0.9991
C.V.	2.94757		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
MACH	1	572.833042	779.50133272	0.735	0.4863
ALT	1	-27.906763	26.05537616	-1.071	0.3197
PIC	1	961.144625	518.38092227	1.854	0.1061
ALPHA	1	750.043663	16155.568176	0.046	0.9643
ALPHASQ	1	-2126.812070	14114.001294	-0.151	0.8845
PICSQ	1	-25.960271	12.86106263	-2.019	0.0833
MDOTSQ	1	-0.027051	0.03917372	-0.691	0.5121

Obs	Dep Var FUEL	Predict Value	Std Err Predict	Lower95% Predict	Upper95% Predict	Residual
1	8203.0	8346.4	110.395	7722.2	8970.6	-143.4
2	7784.0	7979.9	204.937	7234.1	8725.8	-195.9
3	8268.0	8283.3	151.217	7613.0	8953.6	-15.2955
4	8579.0	8205.1	173.681	7505.0	8905.2	373.9
5	8118.0	8233.9	158.449	7554.3	8913.4	-115.9
6	7895.0	7890.3	231.530	7102.1	8678.4	4.7287
7	8130.0	7911.9	160.967	7229.0	8594.8	218.1
8	8393.0	8479.9	173.092	7780.6	9179.1	-86.8736
9	7997.0	7817.7	191.434	7092.2	8543.2	179.3
10	8071.0	7838.6	178.216	7132.2	8545.1	232.4
11	8292.0	8414.9	196.550	7681.8	9148.0	-122.9
12	8190.0	8243.8	137.232	7590.5	8897.0	-53.7504
13	8048.0	8158.7	103.796	7540.9	8776.5	-110.7
14	7911.0	8047.0	154.766	7372.2	8721.8	-136.0

Sum of Residuals 27.51616
Sum of Squared Residuals 402399.7357
Predicted Resid SS (Press) 2153167.3292

Model: MODEL2
NOTE: No intercept in model. R-square is redefined.
Dependent Variable: FUEL

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	7	926652389.08	132378912.73	3869.469	0.0001
Error	7	239477.91551	34211.13079		
U Total	14	926891867			

Root MSE	184.96251	R-square	0.9997
Dep Mean	8134.21429	Adj R-sq	0.9995
C.V.	2.27388		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
MACH	1	899.931461	616.91201063	1.459	0.1880
ALT	1	-51.084435	20.28806379	-2.518	0.0399
PIC	1	555.164873	420.17671073	1.321	0.2280
ALPHA	1	5563.412736	12620.004285	0.441	0.6726
ALPHASQ	1	-6437.671430	11024.625782	-0.584	0.5776
PICSQ	1	-15.985206	10.42460535	-1.533	0.1690
PICPSQ	1	205.962298	87.32001637	2.359	0.0504

Obs	Dep Var FUEL	Predict Value	Std Err Predict	Lower95% Predict	Upper95% Predict	Residual
1	8203.0	8368.1	83.940	7887.8	8848.4	-165.1
2	7784.0	7964.9	151.988	7398.8	8531.0	-180.9
3	8268.0	8097.0	142.695	7544.6	8649.4	171.0
4	8579.0	8397.7	156.027	7825.5	8969.9	181.3
5	8118.0	8176.7	123.484	7650.8	8702.5	-58.6516
6	7895.0	7967.4	128.265	7435.1	8499.6	-72.3818
7	8130.0	8149.3	155.807	7577.5	8721.2	-19.3381
8	8393.0	8364.3	142.099	7812.8	8915.9	28.6758
9	7997.0	7849.6	144.761	7294.2	8405.0	147.4
10	8071.0	7844.6	120.983	7322.0	8367.3	226.4
11	8292.0	8380.7	142.487	7828.6	8932.8	-88.6814
12	8190.0	8269.5	92.302	7780.7	8758.3	-79.5182
13	8048.0	8179.5	79.451	7703.5	8655.5	-131.5
14	7911.0	7855.8	133.155	7316.9	8394.8	55.1552

Sum of Residuals	13.78584
Sum of Squared Residuals	239477.9155
Predicted Resid SS (Press)	1366258.7671

Model: MODEL3
NOTE: No intercept in model. R-square is redefined.
Dependent Variable: FUEL

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	7	926638920.98	132376988.71	3663.386	0.0001
Error	7	252946.02116	36135.14588		
U Total	14	926891867			

Root MSE	190.09247	R-square	0.9997
Dep Mean	8134.21429	Adj R-sq	0.9995
C.V.	2.33695		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
MACH	1	102.961951	619.62076319	0.166	0.8727
ALT	1	-6.991986	23.05783755	-0.303	0.7705
PIC	1	1136.829226	139.72124993	8.136	0.0001
D	1	17.375389	8.54016273	2.035	0.0814
ALPHASQ	1	-7577.532815	3053.4525697	-2.482	0.0421
PICSQ	1	-36.018544	5.71338691	-6.304	0.0004
MDOTSQ	1	0.012339	0.03660211	0.337	0.7459

Obs	Dep Var FUEL	Predict Value	Std Err Predict	Lower95% Predict	Upper95% Predict	Residual
1	8203.0	8280.2	87.571	7785.3	8775.1	-77.1841
2	7784.0	7946.4	155.877	7365.1	8527.7	-162.4
3	8268.0	8198.7	121.958	7664.6	8732.7	69.3450
4	8579.0	8404.8	169.062	7803.2	9006.3	174.2
5	8118.0	8315.7	121.384	7782.4	8849.1	-197.7
6	7895.0	7862.0	182.749	7238.5	8485.6	32.9738
7	8130.0	8054.7	145.136	7489.2	8620.3	75.2868
8	8393.0	8448.9	121.506	7915.4	8982.4	-55.8928
9	7997.0	7713.0	149.209	7141.6	8284.4	284.0
10	8071.0	7972.3	155.606	7391.4	8553.2	98.7408
11	8292.0	8245.5	135.375	7693.7	8797.4	46.4712
12	8190.0	8170.7	95.351	7667.8	8673.5	19.3396
13	8048.0	8173.0	71.895	7692.4	8653.6	-125.0
14	7911.0	8078.0	120.420	7545.9	8610.1	-167.0

Sum of Residuals 15.09726
Sum of Squared Residuals 252946.0212
Predicted Resid SS (Press) 2051958.1201

Model: MODEL4
Dependent Variable: THRUST

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	7	45698053.07	6528293.2957	125.155	0.0001
Error	6	312969.85843	52161.64307		
C Total	13	46011022.929			

Root MSE	228.38924	R-square	0.9932
Dep Mean	15230.07143	Adj R-sq	0.9853
C.V.	1.49959		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	-9830.408896	5100.1114533	-1.927	0.1022
MACH	1	-15210	744.27074445	-20.435	0.0001
ALT	1	609.068150	25.69605142	23.703	0.0001
PIC	1	2443.921487	707.69540040	3.453	0.0136
ALPHA	1	5084.647466	15390.789693	0.330	0.7524
ALPHASQ	1	-7317.429166	13444.542503	-0.544	0.6059
PICSQ	1	-69.709396	18.05144696	-3.862	0.0083
MDOTSQ	1	0.555410	0.03864811	14.371	0.0001

Obs	Dep Var THRUST	Predict Value	Std Err Predict	Lower95% Predict	Upper95% Predict	Residual
1	14972.0	14680.3	112.045	14057.9	15302.8	291.7
2	15029.0	15007.2	200.605	14263.4	15751.0	21.8133
3	12904.0	13262.0	144.160	12601.1	13922.8	-358.0
4	14539.0	14543.1	222.735	13762.5	15323.8	-4.1493
5	17004.0	17134.0	153.632	16460.5	17807.5	-130.0
6	17710.0	17716.8	220.735	16939.6	18494.0	-6.7924
7	14161.0	13932.9	174.476	13229.6	14636.1	228.1
8	12820.0	12701.1	173.610	11999.1	13403.1	118.9
9	18288.0	18288.5	185.713	17568.2	19008.8	-0.5075
10	12855.0	12946.2	190.436	12218.6	13673.9	-91.2238
11	14509.0	14506.9	195.151	13771.8	15241.9	2.1309
12	14308.0	14391.8	135.044	13742.6	15041.0	-83.8131
13	16469.0	16482.1	106.327	15865.6	17098.5	-13.0618
14	17653.0	17628.1	151.042	16958.1	18298.1	24.9142

Sum of Residuals 0
Sum of Squared Residuals 312969.8584
Predicted Resid SS (Press) 1072911.2265

Model: MODEL5
Dependent Variable: THRUST

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	7	45711111.217	6530158.7452	130.642	0.0001
Error	6	299911.71188	49985.28531		
C Total	13	46011022.929			

Root MSE	223.57389	R-square	0.9935
Dep Mean	15230.07143	Adj R-sq	0.9859
C.V.	1.46798		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	-12995	7157.0914986	-1.816	0.1193
MACH	1	-15495	754.32761738	-20.541	0.0001
ALT	1	623.381481	34.81374883	17.906	0.0001
PIC	1	2988.163977	809.58964055	3.691	0.0102
D	1	8.820048	14.40046890	0.612	0.5627
ALPHASQ	1	-6092.486721	5286.9474790	-1.152	0.2930
PICSQ	1	-86.484675	24.93667524	-3.468	0.0133
MDOTSQ	1	0.581526	0.05711261	10.182	0.0001

Obs	Dep Var THRUST	Predict Value	Std Err Predict	Lower95% Predict	Upper95% Predict	Residual
1	14972.0	14682.6	103.811	14079.5	15285.8	289.4
2	15029.0	15036.4	188.093	14321.5	15751.3	-7.4090
3	12904.0	13235.5	150.125	12576.6	13894.5	-331.5
4	14539.0	14552.5	218.583	13787.4	15317.5	-13.4559
5	17004.0	17174.5	169.807	16487.6	17861.5	-170.5
6	17710.0	17700.3	214.955	16941.4	18459.2	9.6868
7	14161.0	13948.6	173.524	13256.1	14641.1	212.4
8	12820.0	12695.9	155.654	12029.3	13362.4	124.1
9	18288.0	18234.9	209.848	17484.6	18985.2	53.0655
10	12855.0	12956.9	187.583	12242.8	13671.0	-101.9
11	14509.0	14496.4	162.045	13820.8	15172.1	12.5708
12	14308.0	14353.4	112.417	13741.1	14965.8	-45.4304
13	16469.0	16499.0	107.112	15892.4	17105.6	-29.9949
14	17653.0	17654.0	156.329	16986.5	18321.6	-1.0332

```
Sum of Residuals                    3.11E-11
Sum of Squared Residuals         299911.7119
Predicted Resid SS (Press)      1439588.8208
```

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Vita

Captain Alan Lach was born on March 21, 1956 in Southbridge, Massachusetts. He graduated from Bay Path Regional Vocational Technical School in Charlton, Massachusetts in 1974 and enlisted in the U. S. Air Force. After achieving the rank of Technical Sergeant in 1982, he was accepted into the Airman's Educationing and Commissioning Program. He graduated cum laude from the University of Utah with a Bachelor of Science in Mechanical Engineering in March 1985. His first commissioned assignment was as Officer in Charge of the Peacekeeper Mechanical Engineering for the 90th Strategic Missile Wing, Francis E. Warren AFB, Wyoming. He was responsible for the operational integration of the Peacekeeper missile system in Minuteman silos. He created and led a special inspection team and was responsible for inspecting and accepting all Peacekeeper missile sites for the U. S. Air Force. In July 1988 he was reassigned as Chief of Mechanical Engineering for Detachment 1, Ballistic Missile Office, Site Activation Task Force at Francis E. Warren AFB. In this capacity he was responsible for follow-on test and evaluation and operational integration of the Peacekeeper missile system as well as deployment planning for the Peacekeeper Rail Garrison program. He entered the School of Engineering, Air Force Institute of Technology, in May 1991.